AS PER GARP FRM PART I
2025 CURRICULUM

TABLE OF CONTENTS

Preface

Welcome to the comprehensive study notes for the Financial Risk Manager (FRM) Part I Exam for the year 2024. The FRM designation, administered by the Global Association of Risk Professionals (GARP), is a globally recognized benchmark for excellence in the field of financial risk management. It signifies a professional's ability to manage risk in today's rapidly evolving financial world.

The purpose of these study notes is to provide candidates with a robust tool aligned with the GARP curriculum, aimed at facilitating a deeper understanding of the fundamental concepts of risk management. Crafted meticulously to cater to the needs of FRM Part I candidates, these notes are your go-to resource for efficient and effective exam preparation.

The FRM Part I Exam is a rigorous four-hour test, consisting of 100 multiple-choice questions. It evaluates a candidate's conceptual knowledge across four key areas: Foundations of Risk Management and Quantitative Analysis, with 20 questions each, and Financial Markets and Products, alongside Valuation and Risk Models, with 30 questions each. Our study notes are designed to cover these areas comprehensively, providing a clear, concise, and easy-to-understand discussion of each topic.

Structured to reflect the GARP curriculum, our notes adopt a step-by-step approach to break down complex concepts into manageable segments. This method ensures that even candidates pursuing self-study can grasp the material effectively. The notes emphasize conceptual understanding, enabling learners to tackle the exam's challenges with confidence.

To excel in the FRM Part I Exam, we recommend focusing on understanding the concepts thoroughly. Practice is key; solving a wide range of questions will help you become familiar with the numerical aspects of the syllabus. Engaging with mock tests at least one month before the exam is crucial for assessing your preparedness and adjusting your study plan as needed. Regular revision and maintaining a positive mindset are also vital strategies for success.

We have taken great care to update these study notes to reflect any changes in the FRM Part I syllabus for 2024, ensuring that they remain relevant and up-to-date with the latest practices in risk management. However, it is important to note that these notes are intended to complement, not replace, the official study materials provided by GARP.

While we have endeavored to make these notes as comprehensive as possible, we encourage candidates to use them alongside the official GARP resources for the best possible preparation.

For any queries or further assistance, feel free to contact us via WhatsApp at +91 9096131868 or email at falconedufin@gmail.com.

We wish you the best of luck in your journey towards achieving the FRM designation and advancing your career in risk management.

Regards

Shashank Wandhe, FRM
Founder - Falcon Edufin

Table of Contents

Reading 01 Measures of Financial Risk

Learning Objectives

- Describe the mean-variance framework and the efficient frontier.
- Explain the limitations of the mean-variance framework with respect to assumptions about return distributions.
- Compare the normal distribution with the typical distribution of returns of risky financial assets such as equities.
- Define the VaR measure of risk, describe assumptions about return distributions and holding period, and explain the limitations of VaR
- Explain and calculate Expected Shortfall (ES), and compare and contrast VaR and ES.
- Define the properties of a coherent risk measure and explain the meaning of each property.
- Explain why VaR is not a coherent risk measure.
- Describe spectral risk measures, and explain how VaR and ES are special cases of spectral risk measures.

1.1 INTRODUCTION

In this reading we will focus on the tools available for measurement(quantification) of financial risk. There are various tools available for measurement of financial risk but in this reading we will mainly focus on VaR (value at risk) and Expected shortfall. In this reading in order to keep discussion simple, we will keep our focus on key concepts (as per Learning objectives). However, there is lot to discuss about VaR which is covered in detail in FRM Part II in around 8 readings.

1.2 VALUE AT RISK INTRODUCTION

Value at risk or VaR is most widely used risk measurement tool. VaR is the statistical measure is the worst possible loss at a specific confidence level over time horizon. Please note, VaR is forecast calculated using historical data or simulated data. As we can not provide any assurance about any forecast, the VaR is forecasted at a given probability known as confidence level.

Let's say daily VaR at 95% confidence level of Mr. A's portfolio is $10 million. First, we will decode this statement to understand VaR in better manner. There are 3 components in this statement-

- **Component 1** - Time horizon: The VaR in current case is daily VaR. This indicates the on any given day maximum loss at 95% confidence level is 10M. VaR can be daily, weekly, monthly or even yearly. If the VaR is monthly VaR, this gives us the maximum monthly loss at given confidence interval.
- **Component 2** – Confidence level: The VaR in the current case is calculated at 95% confidence level. The general meaning of this statement is, we are 95% confident that loss of the portfolio will not exceed $10M. Confidence level is indicating the 'probability' of maximum loss in given time frame. As the confidence level increases VaR will increase. As such there is no restriction on choice of confidence level but calculating VaR at 90%, 95%, 99% and 99.99% is standard. When VaR is calculated for regulatory purposes, regulators provide the confidence level for VaR calculation.
- **Component 3**-Value of VaR: VaR value is the maximum possible loss at given confidence level for given time frame. In this case the VaR of $10M indicates the maximum loss at confidence level. Please note, $10M is not absolute maximum loss, it is maximum loss at given confidence. In risk management, we can not provide absolute assurance about the losses hence we use confidence level.

Interpretation of the statement (very important for exam):

- **Interpretation 1:** There is 95% probability that the loss on portfolio will not exceed $10M on any given day.
- **Interpretation 2:** If we consider 100 days time period, out of 100 days on 95 days loss on portfolio will not exceed $10M and there may be 5 days where loss will exceed $10M. If we consider 1000 days, then in 950 days loss will not exceed $10M and there will be 50 days when loss will exceed $10M.

1.3 CALCULATION OF VAR

The VaR calculation approaches can be categorized in different methods. However, from FRM curriculum standpoint, VaR calculation approaches can be categorized as -

- **Non parametric approach**: In non-parametric approach we use the observation to calculate VaR. We do not require any parameters or assumptions in this approach. In this approach we can use historical data or simulation data for VaR calculation. When the data is not normally distributed, this method is preferred.
- **Parametric Approach:** In this approach, we relay on parameters like standard deviation for VaR calculation. We use equation in parametric approach.

1.3.a non-parametric approach

To calculate the VaR using historical data, we collect the return data of portfolio. For daily VaR calculation, we need daily return data of past few days. Total number of days should be enough to capture potential losses.

Illustration:

Following table provides past 100 days return data of portfolio (Historical data).

4.90%	11.51%	0.40%	21.90%	3.65%	13.20%	14.56%	0.84%	17.56%	9.57%
2.34%	0.01%	-8.08%	3.30%	28.17%	-5.80%	-5.74%	5.54%	5.09%	16.10%
0.46%	2.06%	-4.56%	8.13%	-2.10%	11.82%	12.12%	9.71%	13.08%	-2.24%
22.85%	23.25%	12.70%	-3.18%	14.02%	0.63%	-3.54%	6.08%	9.17%	1.82%
17.76%	-2.91%	4.04%	-8.99%	14.10%	-0.62%	19.95%	-10.84%	1.41%	1.24%
1.82%	9.25%	-5.60%	19.49%	8.03%	13.88%	11.96%	14.40%	26.51%	10.15%
8.78%	-8.10%	9.29%	-1.23%	15.39%	4.13%	3.78%	10.46%	9.57%	18.95%
-4.53%	10.49%	-14.91%	1.46%	13.06%	13.55%	4.49%	21.86%	6.45%	6.75%
15.88%	-6.86%	-7.82%	21.69%	1.08%	11.99%	16.85%	5.45%	15.99%	11.09%
20.07%	17.35%	16.81%	-9.34%	-1.65%	-4.04%	-1.09%	-8.82%	12.85%	9.29%

First step is to sort this data in ascending order. For calculation we need only worst 10 losses hence only 10 worst losses are provided below. We use significance level (1-confidence level to find the maximum loss for VaR calculation).

Lowest Return
-14.91%
-10.84%
-9.34%
-8.99%
-8.82%
-8.10%
-8.08%
-7.82%
-6.86%
-5.80%

The 95% daily VaR can be found using 5th (5% significance level) worst loss which is 8.82%.

The 99% daily VaR can be found using 1st worst loss which is 14.91%.

The 90% daily VaR can be found using 10th worst loss which is 5.80%.

In the above illustration we used historical data for calculation. We can also use simulated data of returns. Simulated data is generated using parameters calculated using past observations. The only difference is in observations and rest of the process is same.

1.3.b Parametric Approach

Parametric approach uses parameters for VaR calculation. Using the same historical data we first need to calculate mean and standard deviation of return which is then used for VaR calculation.

$$\%VaR_{CL\%} = -\mu + Z_{CL\%} X \, \sigma$$

Illustration: Let's assume mean return of 0.2% and standard deviation of 9.5%. To calculate daily VaR at 95% confidence level, we need left tail (one tail) cumulative z value at 5%. Z value for 5% significance is 1.65.

$$\%VaR_{95\%} = -0.002 + 1.65\ X\ 0.95 = 15.65\%$$

We can also calculate VaR at 90% and 99% confidence levels using following z values. You must remember following z values for exam.

Z value for standard confidence level

Confidence level	One tail Z value
90%	1.28
95%	1.65
99%	2.33

In the previous illustration we calculated % VaR. To calculate % VaR into dollar value we need to multiply %VaR with portfolio value.

Illustration: Assume in above case the portfolio size is $100,000. To calculate dollar VaR

$VaR = 100,000 X 15.65% = $15650

1.4 TIME AND CONFIDENCE LEVEL ADJUSTMENT IN VAR

The VaR can be calculated for various time horizon and confidence level. However, if we have VaR for a specific time period and confidence level we can convert it into another time horizon and confidence level. Conversion is very important for VaR. Let's say we calculated VaR using daily return standard deviation which will give us daily VaR. What if we are interested into weekly VaR or monthly VaR? Here we have two solutions, first is to calculate the VaR again using weekly or monthly standard deviation of return. This solution is very time consuming. The second solution is using established principles of time translation of standard deviation we can convert time period of VaR.

1.4.a Time Adjustment

The VaR can be adjusted for time by multiplying or dividing by under root of time. Time is adjusted by under root of time which is similar to time adjustment of standard deviation.

From smaller time to higher time – by Multiplying $\sqrt{time}$

Consider a daily VaR at 95% confidence level of a portfolio is $10M. To convert it into weekly VaR, monthly VaR and annual VaR

- Weekly VaR = $10\ X\ \sqrt{5}$ = 22.36M, assuming 5 trading day in a week
- Monthly VaR = $10\ X\ \sqrt{20}$ = 44.72M, assuming 20 trading day in a month
- Yearly VaR = $10\ X\ \sqrt{250}$ = 158.11M, assuming 250 trading day in a year.

From higher time to smaller time – by dividing $\sqrt{time}$

Consider an annual VaR at 95% confidence level of a portfolio is $160M. To convert it into daily or monthly VaR,

Daily VaR = $\frac{160}{\sqrt{250}}$ = 10.1M

Monthly VaR = $\frac{160}{\sqrt{12}}$ = 46.19M

1.4.b Confidence level adjustment

Consider daily VaR at 95% confidence level of a portfolio is $10 million. To convert VaR from one confidence level from another confidence level, we have to remove the impact of confidence level used for VaR calculation by dividing z value and add impact of new confidence level by multiplying z value.

Illustration

Consider a daily VaR at 95% confidence level of a portfolio is $10M, to calculate VaR at 90% confidence level and 99% confidence level –

Daily 90% VaR = $\frac{10}{1.65}X1.28$ = 7.75M

Daily 99% VaR = $\frac{10}{1.65}X2.33$ = 14.12M

1.5 EXPECTED SHORTFALL

One major problem with VaR is that it does not talk about the losses exceeding VaR. Consider the VaR with 95% confidence level is USD 10M. The risk manager is also interested in knowing the potential losses at 5% times when losses exceed the VaR limit. Expected shortfall is the risk measure that takes into account of expected losses beyond the VaR limit. Expected shortfall is also known as conditional VaR because it is the expected loss conditional that the loss is greater than the VaR. When the losses are normally distributed with mean μ and standard ddeviation σ, the expected shortfall is.

$$\mu + \sigma \frac{e^{-\frac{U^2}{2}}}{(1 - X)\sqrt{2\pi}}$$

Where x is the confidence level and U is the point in the standard normal distribution. From exam standpoint this formula is very difficult, and it is highly unlikely to get question on expected shortfall which requires use of this formula.

Illustration: Consider an example of losses normally distributed with mean of USD 20M and standard deviation of USD 30M. The expected shortfall at 99% confidence level –

$$-20 + 30 \frac{e^{-\frac{2.326^2}{2}}}{(1 - 0.99)\sqrt{2X3.1416}} = USD59.96M$$

Expected shortfall can also be calculated using historical data by simple method. We will use the same illustration used for VaR calculation. At 95% confidence level VaR was 5th lowest value 8.82%. For expected shortfall we take the average of all the values exceeding (in quantity

Lowest Return
-14.91%
-10.84%
-9.34%
-8.99%
-8.82%
-8.10%
-8.08%
-7.82%
-6.86%
-5.80%

ignoring sign) the VaR value.

$$Expected\ shortfall\ at\ 95\%\ CL = \frac{8.99 + 9.34 + 10.84 + 14.91}{4} = 11.02\%$$

Exam note:

- While calculating expected shortfall we do not take the value of VaR cutoff (8.82%). This is the most common mistake committed by students in exam.
- Expected shortfall is also known as conditional VaR or expected tail loss (ETL).

1.5.a Expected shortfall vs VaR

Expected shortfall is better risk measure than the Value at risk due following reason.

- ES provides the estimates of losses beyond the percentile whereas VaR only provides the loss at certain limit.
- ES is coherent risk measure (discussed below) whereas VaR is not coherent risk measure.
- ES is better for portfolio optimization compared to VaR.
- ES is less restrictive in assumptions compared to VaR.

1.6 COHERENT RISK MEASURES

Regulators have historically used VaR to determine the capital banks must keep when internal models are used. For example, the rules introduced for market risk in 1996 based the required capital on the VaR with a ten-day time horizon and a 99% confidence level. Meanwhile, Basel II based credit risk capital on the VaR with a one-year time horizon and a 99.9% confidence level. In 2012, regulators announced their intention to change the market risk capital calculation process by replacing VaR with estimated shortfall and lowering the confidence level to 97.5%.

What is the best risk measure to use when determining capital requirements? Artzner et al. have examined this question theoretically. They first proposed four properties a risk measure should have and the risk measure which follows all of these properties are called **coherent** These are as follows.

1. **Monotonicity:** If (regardless of what happens) a portfolio always produces a worse result than another portfolio, it should have a higher risk measure.

2. **Translation Invariance:** If an amount of cash K is added to a portfolio, its risk measure should decrease by K.
3. **Homogeneity:** Changing the size of a portfolio by multiplying the amounts of all the components by A results in the risk measure being multiplied by A.
4. ***Subadditivity:*** *For any two portfolios, A and B, the risk measure for the portfolio formed by merging A and B should be* no greater than the sum of the risk measures for portfolios A and B.

Question: Is VaR coherent risk measure?

No, VaR is not the coherent risk measure because it ***does not fulfill the subadditivity condition*** because combined VaR of portfolio may exceed the sum of VaR of individual assets under portfolio. Please note, VaR fulfill all the other conditions of coherent risk measure.

Question: Is ES coherent risk measure?

Yes, ES is the coherent risk measures because it fulfills all the conditions of coherent risk measure. Unlike VaR, ES also fulfill the subadditivity condition.

Please note, both the above answers can be show with actual example, however, we are not including proofs here for two main reasons. R1: Not important for exam purpose and R2: Not important in real world (practical life) because these properties are already established, and we don't need to test it again and again. However, if you are interest in example, please refer to GARP curriculum Book 4 Page No 9.

1.7 WEIGHTING AND SPECTRAL RISK MEASURES

In simple terms spectral risk measure is the measure which assigns weight to the percentiles of the loss distribution. VaR at certain confidence level is xth percentile of loss distribution in which all the weight is assigned to xth percentile. ES on the other hand assigns equal weights to losses above VaR level. Hence VaR and ES both are spectral risk measures.

Note: VaR and Es are special cases of spectral risk measures where weights are assigned in a specific manner.

Consider two portfolios A and B with VaR at 95% is $10M for both. However, losses above $10M for portfolio A goes up to $100M whereas for portfolio B its only up to $25. VaR fails to differentiate between these two portfolios, because all the weight is assigned to $10M. If we only consider only VaR risk measure, we might end up assuming both the portfolios are equally risky however it is not the case because A is obviously riskier than the B.

Coherent risk measure requires that the weights are non-decreasing function of the percentile of the loss distribution. In simple language, this means risk measure should assign at least equal weight to higher losses compared to lower. If we consider VaR, all the weight is assigned to xth percentile loss and losses above this percentile are allocated 0% weight which violates coherent risk measure requirement. On the other hand, ES allocates equal weight for all the losses and sticks to coherent risk measure requirement. This property is defined in equation form as

$$weight\ (P1) \geq weigtht(P2) when\ P1 > P2, where\ p1\ and\ p2\ are\ loss\ values$$

There are other measures which are also coherent risk measures. Spectral risk measure is the coherent risk measure which assigns the weight according to risk aversion of investors. In case of VaR and ES, the way in which weights are assigned, it fails to accommodate the investors risk aversion. Spectral risk measure suggests that to achieve this is to make wights proportional to

$$e^{-(1-p)/\gamma}$$

Where p is the percentile and y is a constant reflecting the user's degree of risk aversion. The lower the value of γ, greater the degree of risk aversion.

1.8 MEAN VARIANCE FRAMEWORK

We discussed mean variance framework in Book 1 Reading MPT and CAPM. Hence to avoid the repeated discussion we will only cover key points related to this topic and summary of mean variance framework.

Mean variance framework

- Mean variance framework estimates the risk based on expected return and standard deviation.
- Framework assumes elliptical distribution like normal distribution.
- Two asset portfolio return and risk can be estimated using following formulas.

$$Portfolio\ return = w_1\mu_1 + w_2\mu_2$$

$$Standard\ deviation\ of\ portfolio = \sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\sigma_1\sigma_2 w_1 w_2 \rho_{12}}$$

- Assuming the distribution is normal, portfolios can be chosen based on mean return and standard deviation from the various combination portfolios.
- It is assumed all the investors prefer portfolios on efficient frontier.
- When portfolio is combined with risk free security, optimal portfolios are on the line called capital allocation line. The tangent point of line and efficient frontier is market portfolio, M.
- Under the assumption of available risk-free security, all the investors hold portfolio on the capital allocation line.

1.8.a Limitations of Framework

As the framework depends upon standard deviation, framework is only suitable when returns are normally distributed, however, this assumption is not suitable for risky assets like equity which tends to follow fat tailed (compared to normal) distribution. Framework also assumes the investors take the decision based on mean return and standard deviation, however in reality investors also look for various factors while making investment decisions. The framework assumes the mean, standard deviations and correlations are consistent for all the investors, however in reality all the parameters depend on the choice of time horizon used in historical data used in analysis. For some financial assets distributions are skewed and hence the assumption of normality may result into incorrect inferences.

Reading 02 Calculating and applying VaR

LEARNING OBJECTIVES

- EXPLAIN AND GIVE EXAMPLES OF LINEAR AND NON-LINEAR DERIVATIVES.
- DESCRIBE AND CALCULATE VAR FOR LINEAR DERIVATIVES.
- DESCRIBE AND EXPLAIN THE HISTORICAL SIMULATION APPROACH FOR COMPUTING VAR AND ES.
- DESCRIBE THE DELTA-NORMAL APPROACH FOR CALCULATING VAR FOR NON-LINEAR DERIVATIVES.
- DESCRIBE THE LIMITATIONS OF THE DELTA-NORMAL METHOD.
- EXPLAIN THE FULL REVALUATION METHOD FOR COMPUTING VAR.
- COMPARE DELTA-NORMAL AND FULL REVALUATION APPROACHES FOR COMPUTING VAR.
- EXPLAIN STRUCTURED MONTE CARLO AND STRESS TESTING METHODS FOR COMPUTING VAR, AND IDENTIFY STRENGTHS AND WEAKNESSES OF EACH APPROACH.
- DESCRIBE THE IMPLICATIONS OF CORRELATION BREAKDOWN FOR SCENARIO ANALYSIS.
- DESCRIBE WORST-CASE SCENARIO (WCS) ANALYSIS AND COMPARE WCS TO VAR.

2.1 INTRODUCTION

In the previous reading, we introduced two risk measures: value at risk (VaR) and expected shortfall (ES) and also discussed how they can be calculated. One popular approach is a nonparametric method, where the future behavior of the underlying market variables is determined in a very direct way from their past behavior. This method is known as historical simulation, and it enables the development of many future scenarios. These scenarios can be evaluated using full revaluation (which can be computationally time consuming) or the delta-gamma approach.

Models can also be used to calculate risk measures. For a portfolio that is linearly dependent on underlying market variables, it is relatively easy to calculate the mean and standard deviation of changes in the portfolio value using the mean and standard deviation of the changes in those variables. If we assume the returns on the underlying variables (or changes in their values) are multivariate normal, then changes in portfolio value are also normally distributed and thus calculating VaR is relatively straightforward. This approach is sometimes referred to as the delta-normal model. For a portfolio that is not linearly dependent on the underlying market variables (e.g., because it contains options), the delta-normal model can also be used. However, it is less accurate in this context.

An alternative to historical simulation and the delta-normal model is provided by Monte Carlo simulations. These are like historical simulation, but their scenarios are randomly generated (rather than being determined directly from the behavior of market variables in the past).

2.2 LINEAR VS NON-LINEAR DERIVATIVES

Linear portfolio is linearly dependent on the change in the value of its underlying variables. The stocks or futures/ forwards are linear portfolios because of the linear function between the underlying variables and portfolio value changes. The option is nonlinear derivatives because its payoff depends nonlinearly on the underlying price changes. Hence the portfolio consisting of options are nonlinear portfolios.

2.3 HISTORICAL SIMULATION APPROACH

Historical simulation uses the market variables which impacts the portfolio value. The variables like equity price, interest rates, credit spread, and volatilities are considered. These variables are also known as risk factors. Daily data is collected on the behavior of the risk factors over the past period. These factors are then used to construct the scenarios. Risk factors are divided into two categories.

- % change in past is used to define the % change in future. Eg. the stock prices.
- $ change in past is used to define the $ change in future. Eg. Interest rates and credit spread.

Using these factors, the daily returns are generated. These returns are sorted into highest loss to lowest loss or gains. Now recall the VaR calculation we did in the previous reading using historical data. We use the same approach, like for 95% VaR, selecting the 5th lowest VaR and for Expected shortfall, taking the average of all the losses above VaR value.

What is different in VaR using historical data and historical simulation?

While calculating the VaR using historical data, we simply collect the historical prices and calculate returns which is used for VaR estimations. On the other hand, in historical simulation approach, we identify factors impacting the portfolio/asset, collect the factor data. The factor data is used to simulate the portfolio returns, which is used for VaR estimation. Please note, in both the processes, VaR estimation method is same. The only difference is portfolio return generation process.

2.4 DELTA NORMAL APPROACH

This is an alternative method to historical simulation. Delta normal approach uses the assumption of returns of underlying asset is multivariate normal, changes in portfolio is normally distributed.

To calculate returns in the process discussed above, we need to value the portfolio for each day (assuming calculation is for daily VaR). We can do it using full revaluation method, i.e by considering factors and impact of factors on portfolio for each scenario or day. This is very time consuming. The alternative to this approach is to value portfolio using Greek latters (like delta). Delta is the most commonly used factor which measures the small changes in portfolio with respect to changes in factors.

$$\delta = \frac{change\ in\ portfolio\ value}{change\ in\ factor\ S} = \frac{\Delta P}{\Delta S}$$

Assume the delta of a portfolio with respect to factor is $5000. This means the $1 change factor will result into $5000 change in portfolio value. The change in portfolio value can be measured as

$$\Delta P = \delta \Delta S$$

We can also combine multiple factors using

$$\Delta P = \sum \delta_i \Delta S_i, where\ \delta i\ is\ the\ delta\ of\ each\ factor$$

For linear derivatives portfolio delta provides an exact answer. For non linear portfolio, delta only provides an approximate answer. For non linear portfolio, we can improve accuracy of portfolio changes by using gamma.

For the calculation of VaR, we use the concepts which we learned in previous reading in parametric approach for VaR calculation.

2.5 MONTE CARLO SIMULATION

Another way to calculate VAR and ES is to use Monte Carlo simulation for return generation. This approach is similar to historical simulation the only difference being Monte Carlo simulation generates scenarios by taking random samples from the distributions assumed for the risk factors. Monte Carlo is suitable for both linear and non linear derivatives.

If we assume the risk factors changes have a mutli variate normal distribution, then the procedure is as follows,

- Value the portfolio today using the current value of the risk factors
- Sample once from the multivariate normal probability distribution for change in risk factors with assumed standard deviation and correlation.
- Use the sampled values of the $\Delta risk\ factor$ at the end of the period under consideration.

- Revalue the portfolio using these risk factors value
- Subtract this portfolio value from the current value to determine the loss
- Repeat steps 2 to 5 many times to determine a probability distribution for the loss.

Unfortunately, Monte Carlo simulation is computationally intensive and thus quite slow. The portfolios that must be assessed are often quite large and a full revaluation of them on each simulation trial can be time consuming. As in the case of historical simulation, the computational time can be reduced by using the delta-gamma approach. This is also known as partial simulation.

In the delta-normal approach it is necessary to assume normal distributions for the risk factors. When Monte Carlo simulation is used, any distribution can be assumed for the risk factors providing correlations between the risk factors can be defined in some way. One approach can be to sample from a multivariate normal distribution in a way that reflects historical correlations and then transform the sampled values to the non-normal distributions that are considered appropriate. The transformation is accomplished on a "percentile-to-percentile" basis. If the value sampled from the normal distribution is the p-percentile, it is transformed to the p-percentile of the assumed distribution.

2.6 WORST CASE SCENARIOS

Occasionally, when there are repeated trials, an analyst will calculate statistics for worst-case results. For example, if a portfolio manager reports results every week, he or she might ask what the worst result will be over a period of 52 weeks. If the distribution of the returns in one week is known, a Monte Carlo simulation can be used to calculate statistics for this worst-case result. For example, one can calculate the expected worst-case result over 52 weeks, the 95th percentile of the worst-case result, and so on.

However, these worst-case statistics should not be regarded as an alternative to VaR and ES. They are only appropriate in the (relatively unusual) situation where there are repeated trials. Even then, risk managers should be interested in aggregate results for a whole period rather than the statics of the results over sub-periods.

Reading 03 Measuring and Monitoring Volatility

Learning Objectives

- Explain how asset return distributions tend to deviate from the normal distribution.
- Explain reasons for fat tails in a return distribution and describe their implications.
- Distinguish between conditional and unconditional distributions.
- Describe the implications of regime switching on quantifying volatility.
- Evaluate the various approaches for estimating VaR.
- Compare and contrast different parametric and non-parametric approaches for estimating conditional volatility
- Calculate conditional volatility using parametric and non-parametric approaches.
- Evaluate implied volatility as a predictor of future volatility and its shortcomings.
- Explain and apply approaches to estimate long horizon volatility/VaR, and describe the process of mean reversion according to a GARCH (1,1) model.
- Apply the exponentially weighted moving average (EWMA) approach and the GARCH (1,1) model to estimate volatility.
- Calculate conditional volatility with and without mean reversion.
- Describe the impact of mean reversion on long horizon conditional volatility estimation.
- Describe an example of updating correlation estimates.

3.1 INTRODUCTION

Using historical data, it would be rather simple to estimate a constant volatility. However, in actuality, volatility varies over time. As a result, asset returns are sometimes not normally distributed; rather, they have fatter tails than a normal distribution would suggest. Because these measurements heavily rely on the tails of asset return distributions, this is significant for the calculation of risk measures (such VaR and projected shortfall).

Instead of assuming asset returns are always normal, one approach is to assume they are normal in the presence of known volatility. The daily return is normal with a significant standard deviation when volatility is strong. The daily return is normal with a modest standard deviation when the volatility is minimal. Despite its shortcomings, the conditionally normal model outperforms the constant volatility model.

It is important to monitor volatility in order to get a current volatility estimate in order to apply the conditionally normal model (so that it can, for instance, be used in conjunction with the delta-normal model described in Chapter 2). We provide two methods for doing that in this chapter: the GARCH (1,1) model and the exponentially weighted moving average (EWMA) model. We also go over how volatility can be predicted for durations longer than a day and how correlation can be monitored using the same tools used to monitor volatility.

3.2 DEVIATION FROM NORMALITY

There are three ways in which an asset's return can deviate from normality.

1. The return distribution can have fatter tails than a normal distribution.
2. The return distribution can be non-symmetrical.
3. The return distribution can be unstable with parameters that vary through time.

Either of the first two situations can result from the third situation. For instance, when the volatility parameter varies over time, the return distribution may become unstable. In comparison to the normal distribution, this results in a return distribution with fatter tails. Understanding why volatility varies is simple. Market stress usually causes volatility to rise. Volatility tends to drop when markets settle.

A non-normal distribution can also result from an unstable mean. For a variety of reasons, distributional means might change throughout time. Think about the anticipated return on stocks, for instance. This is the total of the risk premium and the risk-free rate. In actuality, both evolve with time, and hence, the anticipated return does as well. Similar to how the influence of volatility changing over time is highlighted, the impact of the mean changing over time can also be demonstrated. It's interesting to note that a non-symmetrical distribution can result from the union of a stochastic mean and a stochastic volatility.

We will concentrate on the effect of volatility shifting over time for the majority of this chapter. The volatility has a considerably greater impact on daily statistics than the mean return does. In fact, assuming that the mean daily return is zero is not a bad approximation (as we have noted in prior chapters).We will concentrate on the effect of volatility shifting over time for the majority of this chapter. The volatility has a considerably greater impact on daily statistics than the mean return does. In fact, assuming that the mean daily return is zero is not a bad approximation (as we have noted in prior chapters).

3.2.a Conditional vs unconditional normality

It is crucial to distinguish between a model where returns are unconditionally normal and a model where they are conditionally normal in order to comprehend how fat tails are produced by stochastic volatility, or volatility that fluctuates in an unpredictable fashion over time. The probability distribution of the return for each day has the same normal distribution and the same standard deviation under a scenario where returns are unconditionally normal. However, in a model where the return is conditionally normal, the return distribution is consistent every day while the return's standard deviation changes over time. There are times when it is high and times when it is low. An unconditional distribution with fat tails results as a result of this.

Here, it's important to note that we observe the unconditional distribution when gathering data on daily returns (rather than the conditional distribution). For this reason, when the distribution is conditionally normal, we see fat tails. To estimate a conditional distribution for daily return, volatility can be tracked.

3.2.b Regime Switching volatility

It is frequently logical to assume that volatility fluctuates slowly. Famous mathematician Benoit Mandelbrot even said that large changes typically follow by large changes of either sign, while small changes typically follow by small changes. When volatility is high, the price of an asset experiences significant positive and negative changes. Positive and negative changes are significantly less frequent when volatility is minimal. There are tools that can be used to estimate the current value of volatility when volatility fluctuates slowly, as shown by Mandelbrot's assertion (as we shall see) (i.e., to monitor volatility). Mandelbrot's statement does not always hold, though, as volatility can alter suddenly. Volatility could abruptly increase from 1% to 3% per day due to an unforeseen circumstance or government intervention. It can abruptly move back to 1 percent daily when the markets stabilise. Regime switching is the name given to this occurrence. Because they are frequently unplanned, regime shifts provide additional difficulties for risk managers. Because of the altered economic climate, models that had been performing well may abruptly stop doing so.

3.3 MEASURING VOLATILITY

In this reading we will discuss multiple approaches to measure volatility. The simplest one is using standard deviation of historical data for measuring volatility. As we know standard deviation is measured using

$$\sigma = \sqrt{\frac{\sum(r-\bar{r})^2}{n}}$$

While calculating standard deviation using daily data, we can simplify this formula based on two conditions

- Replacing n-1 instead of n to make the standard deviation measure an unbiased estimate
- Average of r = 0 because average daily change in return is approximately zero by using daily. Assume 10% annual return of stock. When we convert it into daily return (avg) the answer 0.10/365 is very minuscule. Hence, this condition it is fair to assume.

The standard deviation after applying above conditions can be calculated using

$$\sigma = retun\ volatility = \sqrt{\frac{\sum r_{n-1}^2}{n-1}}$$

Alternative method for calculating volatility is using absolute return instead of using squared return. In this approach after calculating the difference in return and average of return, we proceed to standard deviation calculation by dropping signs instead of squaring it for removing positive and negative signs impact.

Using standard deviation is not very reliable method for estimating current volatility. This is because if we use huge data for standard deviation calculation then it also captures/ affected by previous data which may not be relevant for recent volatility. Hence, for the current volatility calculation we prefer, more sophisticated methods such as EWMA and GARCH which we will discuss in below.

3.4 EWMA (EXPONENTIALLY WEIGHTED MOVING AVERAGE)

EWMA volatility model is recommended by Riskmetrics group for forecasting of volatility. The closed form equation for EWMA model is

$$\sigma_n^2 = (1 - \lambda)r_{n-1}^2 + \lambda\sigma_{n-1}^2$$

Where, σ_n^2 is recent variance (or forecast variance)

$r_{n-1}^2$is previous days return (square)

$\sigma_{n-1}^2$is the previous days variance

λ is the weight of previous day's variance.

As per the Riskmetrics it is recommended to use weight (lambda) of 0.94 in 1990 which may not be relevant in todays time. For exam purpose lambda will be specifically provided to you in the question but if lambda is not specifically provided then use 0.94.

Illustration: Consider the recent volatility is 2% per day and return is -1%. Hence the current variance is 0.02 X 0.02 = 0.0004 and return square is 0.0001. Assuming the lambda of 0.94, the square volatility forecast is

Recent variance = (1-0.94) X 0.0001 + 0.94 X 0.0004 = 0.000382

Hence the volatility forecast is $\sqrt{0.000382}$ = 1.95%.

Note for Exam: In exam EWMA or GARCH model based questions are simple plug and play questions where you have to use provided inputs in formula and calculate the answer. If you remember these formula then solving question is not a difficult task. There are few thing to note about EWMA model using which you can reconcile your answer in exam,

- Updated volatility will always be closer to previous volatility. Like in above case, previous day volatility is 2% and updated volatility is 1.95%.
- If the previous day return is positive, then updated volatility will be greater than previous day volatility and for negative previous day return, updated volatility will be less than previous day volatility.

3.4.a Choice of lambda

The recommended Lambda of 0.94 may not be suitable for every model. A risk manager may decides to chose different value for lambda, however, this should be done cautiously because too high or low value of lambda might produce vague result. If risk manager decides to use very high lambda then, model will be less responsive to changing volatility. And choosing low lambda will result in high fluctuations in volatility forecasts. There are methods available for determining appropriate lambda for given situations however, these methods are not relevant for our exam.

3.4.b Alternative weight schemes

Multivariate density estimation is another approach used to determine weights in which analysis is carried out to determine which period in the past resembles more to the current period. And weights are allocated to historical data based on relevance to the current period.

3.5 GARCH(1,1) (GENERALIZED AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY)

GARCH is extension of EWMA model. The GARCH model is differs from EWMA in only one aspect and that is the consideration to long run average variance rate. Volatility forecast using GARCH(1,1) model is given by

$$\sigma_n^2 = \alpha r_{n-1}^2 + \beta \sigma_{n-1}^2 + \gamma V_L$$

Where, α, β and γ are weight given to respective terms and term α is similar to (1- λ) *of EWMA and* β is similar to λ in EWMA model.

V_L is the long run variance and $\gamma V_L = \omega$. And

$$V_L = \frac{\omega}{1 - \alpha - \beta}$$

By substituting ω in above equation,

$$\sigma_n^2 = \omega + \alpha r_{n-1}^2 + \beta \sigma_{n-1}^2$$

Exam Note: V term in long run variance is not the squared term like other two terms. When the long run variance is provided in question, use it directly and do not square it because variance itself is squared value and need not be squared. This is most common mistake made by students in exam.

Important points to note in GARCH model

- Sum of weight α, β and γ = 1
- α + β <= 1 to keep the model stable, because total of all three weights is equal to 1.
- γ = (1 - $\alpha - \beta$)
- $V_L = \frac{\omega}{1-\alpha-\beta}$

Illustration

ω **=** 0.000003

α = 0.12

$\beta = 0.87$

Long run average variance = 0.0003

Previous day volatility is 2% and return is -3%,

The updated variance is equal to

$$\sqrt{0.000003 + 0.12(-0.03)^2 + 0.87\ X\ 0.02^2} = 2.14\%$$

3.6 MEAN REVERSION

The difference between EWMA and GARCH (1,1) is that in GARCH (1,1), the VL term provides a "pull" toward the long run average mean. In EWMA, however, there is no pull because this term is not included. When volatility is above the long run average mean, it tends to be pulled down toward it. When the volatility is below the long run average mean, it tends to get pulled up toward it. Of course, this is just a tendency. The observed returns will lead to ups and downs superimposed on the average path given by the VL term. This "pull" is referred to as mean reversion. GARCH(1,1) incorporates mean reversion whereas EWMA does not. It is important to note that mean reversion is a reasonable property for some market variables, but not for others. If the market variable is the price of something that can be traded, it should not exhibit a predictable mean reversion because otherwise there would be a market inefficiency which can be exploited for profit.

3.7 IMPLIED VOLATILITY (For Volatility Measurement)

No discussion of volatility would be complete without mentioning implied volatility. The implied volatility is the volatility implied from options prices. We will discuss this in some detail in later chapters. At this stage we note the value of a call or put option depends on volatility. As volatilities increase, the value of the option increases. This is what enables a volatility to be implied from an options price. Implied volatilities are forward looking, whereas the volatilities calculated from historical data (e.g., EWMA, GARCH, or some other method) are backward looking. When market participants are pricing options, they are thinking about what volatility will be in the future, not what it has been in the past. Evidence indicates that implied volatilities give better estimates of realized volatility than historical volatilities. The implied volatility of an option is usually expressed as a volatility per year. It can be converted to a volatility per day by dividing by the square root of 252.

The implied volatility of a one-month option gives an indication of the average volatilities expected over the next month, the implied volatility of a three-month option gives an indication of what volatilities are expected to be over the next three months, and so on. Because volatilities exhibit mean reversion, we do not expect implied volatilities to be the same for options of all maturities.

3.8 USE OF EWMA MODEL FOR CORRELATION MEASUREMENT

It is important for risk managers to monitor correlations as well as volatilities. When the delta-normal model is used to calculate VaR or expected shortfall for a linear portfolio, correlations between daily asset returns (as well the daily standard deviations of asset) are needed. To update correlation estimates, we can use rules similar to those we use for updating volatility estimates. Just as updating rules for volatility work with variances, the updating rules for correlations work with covariances. If the mean daily returns are assumed to be zero, the covariance of the returns between two variables is the expectation of the product of the returns. The EWMA model for updating the covariance between return X and return Y is

$$Cov_n^2 = \lambda\, Cov_{n-1} + (1-\lambda) X_{n-1}\, Y_{n-1}$$

Illustration

Suppose volatility of X and Y are estimates for day n-1 as 1% and 2% per day. Coefficient of correlation has be estimated as 0.20. The covariance can be calculated as

$$Cov_{n-1} = 0.2\, X\, 0.01\, X\, 0.02 = 0.00004$$

Using the above formula, covariance forecast is (assuming lambda of 0.94 and 2% return on previous day for X and Y both)

$$Cov_n^2 = 0.94\, x\, 0.00004 + 0.06\, X\, 0.02\, X\, 0.02 = 0.0000616$$

Using above covariance estimate we can get correlation estimate

$$\frac{0.0000616}{0.01086\, X\, 0.02} = 0.28$$

Reading 04 External and Internal Credit Rating

Learning objectives

- Describe external rating scales, the rating process, and the link between ratings and default.
- Describe the impact of time horizon, economic cycle, industry, and geography on external ratings.
- Define and use the hazard rate to calculate unconditional default probability of a credit asset.
- Define recovery rate and calculate the expected loss from a loan.
- Explain and compare the through-the-cycle and at-the point internal ratings approaches.
- Describe alternative methods to credit ratings produced by rating agencies.
- Compare external and internal ratings approaches.
- Describe and interpret a ratings transition matrix and explain its uses.
- Explain the potential impact of ratings changes on bond and stock prices.
- Explain historical failures and potential challenges to the use of credit ratings in making investment decisions.

4.1 INTRODUCTION

Rating system: Rating in simple terms is the grading system which is designed to provide the credit risk of an organization, country, or security. Credit rating provides the answer to the question, "how likely is an entity to default?". For example, organization with AAA credit rating is safer from credit risk standpoint compared to organization with BBB credit rating. Ratings are provided by credit rating agencies on the rating scale.

Important point to note here is, Credit rating of a company can be different from the credit rating of security issued by same company. A company with BBB credit rating may issue security with AA rating. This can be achieved by structuring security in such a manner, which reduces the credit risk for the investors investing in this security.

Rating agencies: Rating agencies are companies who provide ratings based on their assessment. The rating provided by one agency might differ from another, however, in most of the cases on the similar range of ratings are provided. The most well-known credit rating agencies are US based agencies which are Moody's, Standard and Poor's (S&P), and Fitch. There are also many smaller rating agencies (e.g., DBRS) throughout the world. Credit rating agencies have been doing this for more than 100 years with good track record but with few exceptions. Once such exception is 2007-2008 subprime crisis where, credit rating agencies where they played major role in crisis by providing excessively high ratings to subprime products that subsequently defaulted. One of the consequences of this is that, now The Dodd-Frank act requires rating agencies to apply more transparent process their ratings.

Regulators on credit rating (Trust Issues): Credit ratings have been widely used by regulators. Historically, banks were prohibited from investing in firms with low credit ratings. Today, the Basel Committee uses credit ratings when determining credit risk capital for banks. Since the 2007-2008 crisis, however, the United States has indicated it does not want to do this (presumably because it no longer trusts the ratings). For some capital determinations, two alternative calculations have been specified by the Basel Committee: one for countries that are prepared to use external ratings and one for countries that are not.

The reputations of rating agencies have suffered since the crisis because of their poor performance in rating structured products. However, their *performance in rating bonds and money market instruments has been generally good*, and many risk managers still rely on these ratings.

External ratings can be of limited use to risk managers because rating agencies (usually) only look at firms with publicly traded bonds or money market instruments. Firms that fund themselves with bank borrowing and do not issue debt instruments are often not assessed by rating agencies. Because many of their borrowers do not have external ratings, banks have developed their own internal rating systems to help them make lending decisions. These rating systems are structured like those of the major rating agencies.

4.2 RATING SCALES

An external credit rating is usually an attribute of an instrument issued by an entity (rather than of the entity itself). A bond rating can depend on various factors (e.g., collateral, the term of the instrument, and so on), but often an agency will give all instruments issued by an entity the same rating. As a result, bond ratings are often assumed to be attributes of the entity rather than of the bond itself.

Ratings agencies use different scales for bonds and money market instruments (maturity less than one year). The ratings for bonds are termed long-term ratings, whereas those for money-market instruments are termed short-term ratings. We now consider the long-term and short-term scales used by rating agencies.

4.2. a Long Term Rating

	Moody's Rating	S&P Rating & Fitch
Range of ratings	Aaa(Highest) – C(Lowest)	AAA(Highest) – C
Modifiers (are used to provide finer rating measure). Please note AAA and Aaa and lowest C are not subdivided by both Moody's and S&P.	Use of numeric: For example ratings for Aa category is subdivided into Aa1, Aa2, and Aa3.	Use of sign: For similar category A+, A and A- are used.
Junk bonds (non investment grade) are provided ratings below	Baa3	BBB-

4.2.b Short Term Rating

The rating agencies have different ways of rating money market instruments. Moody's uses P-1, P-2, and P-3 as its three prime rating categories. Instruments rated P-1 are considered to have a superior ability to repay short-term obligations, instruments rated P-2 have a strong ability to do so, and instruments rated P-3 have an acceptable ability to do so. These can be thought of as the investment grade ratings. The rating NP denotes nonprime and can be thought of as a non-investment grade rating.

The S&P rating category corresponding to P-1 is divided into two: A-1 + and A-1. The categories equivalent to P-2 and P-3 are A-2 and A-3 (respectively). Meanwhile, S&P has three lower rating categories: B, C, and D. Fitch subdivides its ratings in a similar way to S&P, with its ratings being F1 +, F1, F2, F3, B, C, and D.

4.2.c Performance of Credit Ratings

The performance of the ratings is tested using the historical data on probability of default of a company provided by agency and its subsequent default. Following table provides the default rates produced by S&P for bonds. It shows the cumulative probability that an issuer that starts with given rating will default within the given period (say within one year).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
AAA	0.00	0.03	0.13	0.24	0.35	0.45	0.51	0.59	0.65	0.70	0.73	0.76	0.79	0.85	0.92
AA	0.02	0.06	0.12	0.22	0.32	0.42	0.51	0.59	0.66	0.73	0.80	0.86	0.92	0.98	1.04
A	0.06	0.14	0.23	0.35	0.49	0.63	0.81	0.96	1.12	1.28	1.43	1.57	1.71	1.83	1.98
BBB	0.17	0.46	0.80	1.22	1.64	2.05	2.41	2.76	3.11	3.44	3.79	4.06	4.32	4.59	4.87
BB	0.65	2.01	3.63	5.25	6.78	8.17	9.36	10.43	11.38	12.22	12.92	13.56	14.13	14.63	15.17
B	3.44	7.94	11.86	14.95	17.33	19.26	20.83	22.07	23.18	24.21	25.08	25.73	26.31	26.87	27.43
CCC/C	26.89	36.27	41.13	43.94	46.06	46.99	48.20	49.04	49.80	50.44	50.96	51.51	52.16	52.72	52.80
Investment Grade	0.09	0.25	0.43	0.66	0.90	1.14	1.36	1.56	1.77	1.96	2.16	2.32	2.48	2.63	2.80
Speculative Grade	3.66	7.13	10.12	12.56	14.55	16.18	17.55	18.69	19.70	20.62	21.39	22.02	22.60	23.13	23.65
All Rated	1.48	2.91	4.16	5.21	6.08	6.82	7.44	7.97	8.44	8.88	9.26	9.58	9.87	10.13	10.41

The above table shows an issuer with a rating of AA has 0.02% probability of defaulting within a year. Using above table we can calculate the probability of bond defaulting during a future year by subtracting successive numbers. For example probability that B rated bond will default during 5th year is 2.38% (17.33 – 14.95). Using the same method we can convert the above table into unconditional default probability table which is given below. This table shows the probability of default during the year.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
AAA	0.00	0.03	0.10	0.11	0.11	0.11	0.06	0.08	0.06	0.06	0.03	0.03	0.03	0.07	0.06
AA	0.02	0.04	0.07	0.10	0.10	0.11	0.10	0.08	0.07	0.08	0.08	0.06	0.07	0.07	0.06
A	0.06	0.09	0.10	0.13	0.15	0.16	0.19	0.17	0.18	0.18	0.16	0.16	0.16	0.14	0.17
BBB	0.18	0.33	0.37	0.45	0.45	0.46	0.39	0.38	0.38	0.37	0.40	0.32	0.31	0.31	0.33
BB	0.72	1.52	1.78	1.78	1.65	1.52	1.29	1.15	1.01	0.91	0.73	0.65	0.58	0.51	0.54
B	3.76	4.80	4.10	3.21	2.45	2.00	1.64	1.27	1.14	1.06	0.91	0.69	0.61	0.57	0.59
CCC/C	26.78	9.10	5.08	3.10	2.36	0.96	1.18	0.96	0.86	0.65	0.52	0.55	0.71	0.56	0.00
Investment Grade	0.10	0.17	0.19	0.25	0.25	0.25	0.24	0.22	0.22	0.22	0.22	0.18	0.18	0.17	0.19
Speculative Grade	3.83	3.65	3.15	2.57	2.09	1.72	1.44	1.20	1.06	0.96	0.80	0.66	0.60	0.54	0.53
All Rated	1.52	1.47	1.28	1.08	0.90	0.77	0.65	0.55	0.50	0.46	0.40	0.33	0.31	0.28	0.28

Error note: Above table is taken from GARP curriculum book which shows some wrong values. For example as per our calculation, for B rated bond 5th year probability should be 2.38 whereas table shows 2.45. I used above table just for example, please do not refer to values given in above table for following calculations.

Another way of presenting the probability of default during a year is conditional probability which is given below. This table provides the probability of default if firm survives in the previous years. The probability that firm survives till 4th year is 85% (100% - 14.95%). Please note 14.95% is the cumulative default probability by the end of 4th year. We also know that the probability of firm will default in 5th year is 2.38%. Hence, probability that firm will default during the 5th year, conditional upon no default in earlies years is 2.80 = (0.0238/0.8505)

Formula used above

$$P(Default\ |no\ earlier\ defualt) = \frac{P(Default\ during\ year)}{P(No\ default\ during\ previous\ years)}$$

Using this method we can get the conditional default probability table (given below)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
AAA	0.00	0.03	0.10	0.11	0.11	0.11	0.06	0.08	0.06	0.06	0.03	0.03	0.03	0.07	0.06
AA	0.02	0.04	0.07	0.10	0.10	0.11	0.10	0.08	0.07	0.08	0.08	0.06	0.07	0.07	0.06
A	0.06	0.09	0.10	0.13	0.15	0.16	0.19	0.17	0.18	0.18	0.16	0.16	0.16	0.14	0.17
BBB	0.18	0.33	0.37	0.45	0.46	0.47	0.40	0.39	0.39	0.38	0.42	0.33	0.32	0.33	0.35
BB	0.72	1.53	1.82	1.85	1.75	1.64	1.42	1.28	1.14	1.04	0.84	0.76	0.68	0.60	0.64
B	3.76	4.99	4.48	3.68	2.91	2.45	2.06	1.63	1.48	1.40	1.22	0.94	0.84	0.79	0.82
CCC/C	26.78	12.43	7.92	5.25	4.22	1.79	2.24	1.87	1.70	1.31	1.06	1.14	1.48	1.19	0.00
Investment Grade	0.10	0.17	0.19	0.25	0.25	0.25	0.24	0.22	0.22	0.22	0.22	0.18	0.18	0.17	0.20
Speculative Grade	3.83	3.80	3.40	2.88	2.41	2.03	1.74	1.47	1.32	1.21	1.02	0.85	0.78	0.71	0.70
All Rated	1.52	1.49	1.32	1.13	0.95	0.82	0.70	0.60	0.54	0.50	0.44	0.36	0.34	0.31	0.31

4.2.d Hazard Rate:

Hazard rate also known as default intensity, is the rate of default at time t. Hazard rates are used to calculate the unconditional default probability. Lets assume the constant hazard rate of 1% per year. Then the probability of default by the end of the 3rd year is

$$Cumulative\ default\ probaility = 1 - e^{Hazard\ rate\ X\ year}$$

$$1 - e^{-0.01\ X\ 3} = 2.96\%$$

Similarly, we can calculate unconditional default probability. The probability of default occurring during the fourth year is

$e^{-0.01\ X\ 3} - e^{-0.01\ X\ 4} = 0.966\%$

And conditional probability of default in 4th year is

0.00966/(1-0.0296) = 0.00995

4.3 RECOVERY RATES

Recovery rate(RR) is the rate of recovery based on partial payment made by the firm in case of bankruptcy. If banks outstanding loan to firm A is $1,000,000 and it is expected that the bank will be able to recover $200,00 then the recover rate is 20% for firm A. The loss given default is the amount which is not recovered in case of firm default. Hence

Loss Given Default(LGD) = (1- RR)

To calculate the expected loss from a loan over a certain period

Probability of default(PD) X LGD or

Probability of default(PD) X (1-RR)

Recovery rates are negatively correlated to default rates. During recession, the default rate on the bonds were high and recovery rates were low.

4.4 CREDIT SPREAD AND RISK PREMIUMS

Assume the expected loss on the bond is 0.984% of the bonds principle in next 5 years. In this case bank should charge 0.20%(0.984%/5) yearly over the risk free rate to take the effect of credit risk. The extra interest charged over the risk free rate is known as credit spread. This is actuarial compensation necessary for credit risk. However, bond holders need a risk premium in addition to the actual compensation to take the effect of systemic risk and variability of risk in coming year. A risk premium can be as high as 1% for actuarial risk premium of 0.20% in some cases.

4.5 RATING PROCESS

Rating agencies rate publicly traded bonds and money market instruments. An instrument is typically first rated when it is issued; the rating is also reviewed periodically (usually at least every 12 months). The rating, based on a mixture of analysis and judgement, is only provided when there is information of sufficient quality for the rating agency to form an opinion.

To quote S&P, "[t]he analysis generally includes historical and projected financial information, industry and/or economic data, peer comparisons, and details on planned financings." In addition, the analysis is based on qualitative factors, such as the institutional or governance framework. A meeting with management is also commonly undertaken.

The fee for the rating is paid by the firm being rated. This is generally a few basis points applied to the notional amount of the bond. If a firm chooses not to pay the fee, the rating agency may not issue a rating. This fee arrangement has been criticized because the rating agency is paid by the issuer even though the product it provides is used by the purchaser. It would make sense for the purchaser to pay, but this is organizationally difficult because of what is termed the "free rider" problem; once one investor has bought the rating, others may obtain it from that investor for free.

It is sometimes argued that when the issuer is paying, the rating agency will be inclined to assign a rating that the issuer thinks it deserves. The counterargument to this is that the rating agency relies on its reputation and therefore will not hesitate to give a low rating when it is warranted.

4.5.a Outlook and watchlists

In addition to the ratings themselves, rating agencies provide what are termed outlooks. These are indications of the most likely direction of the rating over the medium term. A positive outlook means that a rating may be raised, a negative outlook means that it may be lowered, and a stable outlook means it is not likely to change. A developing (or evolving) outlook means that while the rating may change in the medium term, the agency cannot (as of yet) determine the direction of this change.

Placing a rating on a watchlist indicates a relatively short-term change is anticipated (usually within three months). Watchlists can be positive (indicating a review for a possible upgrade) or negative (indicating a review for a possible downgrade).

4.5.b Rating stability

Rating stability is an important objective for rating agencies. One reason for this is that bond traders are major users of ratings. Often, they are subject to rules governing the credit ratings of the bonds they hold (e.g., some bond portfolio managers are only allowed to hold investment grade bonds). If ratings changed frequently, bond traders might have to trade more frequently (and incur high transaction costs) to maintain the required bond rating distributions in their portfolios.

Another reason for rating stability is that ratings are used in financial contracts and (in some countries) by financial regulators. Frequent changes in ratings would cause problems. For example, a bond rated A- might be acceptable as collateral in a contract, whereas one rated BBB+ is not. If a bond's rating switched frequently between these two rating categories, it would create difficulties in the administration of the underlying contract.

As a result of this need for stability, ratings only change when rating agencies believe there has been a long-term change in a firm's creditworthiness.

4.5.c Through the Cycle Vs Point in Time

Economies are cyclical and vary from periods of high growth to periods of lower growth (or even contraction). Furthermore, a firm's probability of default changes in tandem with economic Conditions Rating agencies must therefore decide whether to rate firms "through-the-cycle" or at a "point-in-time." A through-the-cycle rating tries to capture the average creditworthiness of a firm over a period of several years and should not be unduly affected by ups and downs in overall economic conditions. By contrast, a point in time rating is designed to provide the best current estimate of future default probabilities. In theory, a through-the-cycle estimate will understate the probability of default during the down part of the economic cycle and overstate it during the up part of the cycle.

Consistent with their desire to produce stable ratings, rating agencies produce through-the-cycle estimates. It is therefore not always the case that ratings worsen when the economy is doing poorly (or improve when the economy is doing well). Sometimes, rating users adjust ratings produced by rating agencies so that they are converted from through-the-cycle to point-in-time. To do this, it is necessary to find a way to index the health of the economy and then apply this measure to the through-the-cycle ratings. The index is applied in such a way that ratings increase when the economy is doing well and decrease when it is doing poorly. These adjustments must be calibrated to empirical data on defaults at various times during the economic cycle.

4.5.d Industry and Geographic Consistency

Rating agencies use the same scales to characterize default risk across different industries and different countries. An important question is whether ratings are consistent. For example, does a BBB+ rating for a firm in a certain industry in California mean the same as a BBB+ rating for a firm in different industry in Germany?

Moody's, S&P, and Fitch are based in the United States and much of the information they report is based on U.S. data. The rating history for firms outside the United States is shorter than for firms in the United States. As a result, it is sometimes difficult to determine whether ratings for non-U.S. firms are consistent with those of U.S. firms. S&P provides statistics like those in Table (Cumulative default) separately for firms in the United States, Europe, and emerging markets. The U.S. data considers default experience over a 15-year period as in Table (of cumulative default), the European data considers default experience for a seven-year period, and the emerging markets data considers default experience over just five years. Table given below compares the five-year default percentages for the three groups. It shows that a rating for a European firm has historically been better than the same rating for a U.S. firm. This is particularly true for firms with investment grade ratings. Indeed, the probability of default for a European firm with an investment grade rating has been around one-third of that for a U.S. firm. The statistics for emerging markets firms show a different pattern from those for European and U.S. firms. Those rated AAA, AA and A had a very low five-year default probability. However, firms rated BBB fared slightly worse than U.S. firms and much worse compared to European firms. Because rating agencies are continuously striving for geographic consistency, we do not necessarily expect these performance differences to be the same in the future as they have been in the past.

There is less available data on the consistency of ratings across industries. In the past, it has been true that banks with a given rating show higher default rates than non-financial corporations with the same rating. Also, there has been less agreement among different rating agencies for banks than for other firms. Again, it is worth stressing that rating agencies strive for consistency and that extrapolating from past data is dangerous. Differences observed in the past may not always be observed in the future.

Initial Rating	U.S. Firms	European Firms	Firms in Emerging Markets
AAA	0.41	0.00	0.00
AA	0.43	0.20	0.00
A	0.69	0.26	0.04
BBB	1.92	0.56	2.24
BB	7.89	3.71	5.26
B	18.70	12.43	12.10
CCC/C	51.42	43.37	26.32
Investment Grade	1.12	0.34	1.45
Speculative Grade	16.47	10.14	9.35
All rated	7.47	2.62	5.87

4.6 ALTERNATIVE METHODS TO RATING (DETAILED DISCUSSION IS COVERED IN FRM PART II)

The ratings produced by credit rating agencies such as Moody's, S&P, and Fitch can be contrasted with the information provided by organizations such as KMV (which is now part of Moody's) and Kamakura. These organizations use models to estimate default probabilities and provide the output from these models to clients for a fee. These models can include factors such as:

- The amount of debt in the firm's capital structure,
- The market value of the firm's equity, and
- The volatility of the firm's equity.

In the simplest version of the model, default can occur at just one future time. The default happens if the value of the assets falls below the face value of the debt repayment that is required at that time. If V is the value of the assets and D is the face value of the debt, the firm defaults when V 6 D. The value of the equity at the future time is

max(V-D, 0)

This shows that the equity is a call option on the assets of the firm with a strike price equal to the face value of the debt. The firm defaults if the option is not exercised. The probability of this can be calculated from standard option pricing theory (which will be covered in later chapters).

KMV and Kamakura provide point-in-time estimates and do not have the stability objective of rating agencies. It can be argued the output from these models responds to changing circumstances much more quickly than the ratings provided by agencies. Equity prices, which are a key input to their models, are continually changing to reflect the latest information. Ratings, on the other hand, are only reviewed periodically.

4.7 INTERNAL RATINGS

Banks and other financial institutions develop their own internal rating systems based on their assessment of potential borrowers. They typically base their ratings on several factors (e.g., financial ratios, cash flow projections, and an assessment of the firm's management). In general, each factor is scored, and then a weighted average score is calculated to determine the overall final rating.

It is important for banks to develop their own internal rating procedures for several reasons. First, external ratings are not always available. Second, regulatory credit risk capital depends on probabilities of defaults (PDs). Finally, the accounting standard IFRS 9 (and its FASB counterpart) require banks to take default probabilities into account when loans are valued on the balance sheet.

Like external ratings, internal ratings can be either through-thecycle or point-in-time. There is a tendency for them to be point in- time, but through-the-cycle ratings may be more relevant for relatively long-term lending commitments. It can be argued that regulators should encourage banks to use through-the-cycle ratings (e.g., by insisting that through-the-cycle probabilities be used for the determination of regulatory capital). This is because point-in-time ratings are pro-cyclical (i.e., they may accentuate economic cycles). During bad economic conditions, point-in-time probabilities of default increase and banks become less inclined to lend. This makes it difficult for firms to fund working capital and fixed assets; this in turn can cause economic conditions worsen further. During good economic conditions, the reverse happens and economic conditions are helped by an easing of credit.

Banks must back-test their procedures for calculating internal ratings. This typically requires at least ten years of data and involves producing something equivalent to Table of cumulative default probabilities. If the default statistics show that firms with higher ratings have performed better than those with low ratings, then a bank can have some confidence in its rating methodology.

Some banks are currently trying to automate their lending decisions using machine learning. With this approach, an algorithm is given a great deal of historical data on firms and whether they have defaulted. This is used to come up with a rule for distinguishing between those firms that default from those that do not. Arguably, the first attempt to do something like this was proposed by Altman in 1968. He developed what has become known as the Z-score. Using a statistical technique known as discriminant analysis, he looked at the following ratios:

- X1: Working capital to total assets,
- X2: Retained earnings to total assets,
- X3: Earnings before interest and taxes to total assets,
- X4: Market value of equity to book value of total liabilities, and
- X5: Sales to total assets.

For publicly traded manufacturing firms, the Z-score was:

$$Z = 1.2X1 + 1.4X2 + 3.3X3 + 0.6X4 + 0.999X5$$

A Z-score above 3 indicated that the firm was unlikely to default. As the Z-score was lowered, the probability of default increased to the point where a firm with a Z-score below 1.8 had a very high probability of defaulting.

Today's machine learning algorithms use far more than five input variables and far more data than that used by Altman. Furthermore, the discriminant function does not have to be linear.

4.8 RATING TRANSITION MATRIX

In addition to the information presented in Table of cumulative default probabilities, rating agencies produce rating transition matrices. These are tables showing the probability of a bond issuer migrating from one rating category to another during a one-year period. Following table shows the one-year rating transitions produced by S&P in its 2018 study (based on data taken from 1981 to 2018).

	AAA	AA	A	BBB	BB	B	CCC/C	D	NR
AAA	86.99	9.12	0.53	0.05	0.08	0.03	0.05	0.00	3.15
AA	0.50	87.06	7.85	0.49	0.05	0.06	0.02	0.02	3.94
A	0.03	1.69	88.17	5.16	0.29	0.12	0.02	0.06	4.48
BBB	0.01	0.09	3.42	86.04	3.62	0.46	0.11	0.17	6.10
BB	0.01	0.03	0.11	4.83	77.50	6.65	0.55	0.65	9.67
B	0.00	0.02	0.08	0.17	4.93	74.53	4.42	3.44	12.41
CCC/C	0.00	0.00	0.11	0.20	0.59	13.21	43.51	26.89	15.50

According to Table , an issuer that has just been giving an A rating has an 88.17% probability of being A-rated one year later. On the upside, it also has a 1.69% chance and a 0.03% chance of being upgraded to AA and AAA (respectively). On the downside, it has a 5.16% chance and a 0.06% chance of being downgraded to BBB and defaulting (respectively). The NR column indicates the probability that a firm is no longer rated at the end of a year. For analysis, it is often necessary to proportionally allocate the NR number to the other rating categories.

The table shows that investment grade ratings are more stable than non-investment grade ratings. For example, a AAA-rated firm has an 86.99% chance of staying AAA; a BB rated firm only has a 77.50% chance of staying BB. Meanwhile, firms in the CCC/C category only have a 43.51% chance of keeping their ratings during the year.

If we assume rating changes in successive years are independent (i.e. what happens in one year is not influenced by what happened in the previous year), we can calculate a transition matrix for n years using the transition matrix for one year (this involves matrix multiplication). The actual multi-year transition matrices (as reported by the rating agencies) are not quite the same as those calculated using this independence assumption. This is because of what is referred to as the ratings momentum phenomenon. If a firm has been downgraded in one year, it is more likely to be downgraded the next year. If a firm has been upgraded one year, it is more likely to be upgraded the next year.

Rating transition matrices are calculated for internal as well as external ratings. One test of ratings is whether rating transitions remain roughly the same from one year to the next. Research conducted several years ago suggests there are some differences across sectors, especially for investment grade issuers.8 Transition matrices also seem to depend on the economic cycle. Specifically, downgrades increase significantly during recessions (this is despite the fact the ratings are designed to be through-the-cycle).

4.9 CREDIT RATING CHANGES ANTICIPATION

An interesting question is whether ratings have information content. It is possible that when a rating is moved down, new information is being provided to the market so that both the stock and bond prices decline while credit default swaps spreads increase. It is also possible that the market has anticipated the information, making the rating agency a follower rather than a leader.

Researchers who have investigated this question have produced mixed results. Most agree that the stock and bond markets' reactions to downgrades are significant. This is particularly true when the downgrade is from investment grade to noninvestment grade. However, the market's reaction to upgrades is much less pronounced. Part of the reason why downgrades impact prices while upgrades do not is that downgrades (particularly those from investment to non-investment grade) affect the willingness of investors to hold bonds. Also, these firms may have entered into contracts involving rating triggers, and a downgrade may have negative implications for them.

Hull, Predescu, and White (2004) look at the impact of rating changes on credit default swap spreads.9 They examined at outlooks and watchlists as well as rating changes. They found watchlist reviews for a downgrade contain significant information, but downgrades and negative outlooks do not. Positive rating events were much less significant. Generally, credit default swap changes seem to anticipate rating changes.

Indeed, they found credit spread changes provide helpful information in estimating the probability of negative credit rating changes. They found that 42.6% of downgrades, 39.8% of reviews for downgrades, and 50.9% of negative outlooks come from the top quartile of credit default swap spread changes.

4.10 THE RATING OF STRUCTURED PRODUCTS

During the run-up to the 2007–2008 crisis, rating agencies became much more involved in the rating of structured products created from portfolios of subprime mortgages. A key difference between rating structured products and the traditional business of the rating agencies (i.e., rating regular bonds and money market instruments) is that the rating of a structured product depends almost entirely on a model. The rating agencies were quite open about the models they used; S&P and Fitch based their ratings on the probability that the structured product would give a loss, while Moody's based its ratings on expected loss as a percent of the principal. Unfortunately, the inputs to their models (particularly the correlations between the defaults on different mortgages) proved to be too optimistic, and their ratings of structured products created from other structured products proved to be questionable.

Once the creators of structured products understood the models used by rating agencies, they found that they could design the structured products in a way that would achieve the ratings they desired. In fact, they would present plans for structured products to rating agencies and get advanced rulings on ratings. Products that did not receive the desired rating were adjusted until they did.

Rating agencies found the work they were doing on structured products to be very profitable and they (perhaps) were not as independent as they should have been. As is well known, many of the structured products created from mortgages defaulted during the 2007–2008 crisis period and the reputation of rating agencies suffered as a result. As already mentioned, rating agencies are now subject to more oversight than before and (at least in the United States) are no longer used by bank supervisors to determine regulatory capital.

Reading 05 Country Risk: Determinants, Measures, and Implications

Learning Objectives

- Identify sources of country risk.
- Explain how a country's position in the economic growth life cycle, political risk, legal risk, and economic structure affect its risk exposure.
- Evaluate composite measures of risk that incorporate all major types of country risk.
- Compare instances of sovereign default in both foreign currency debt and local currency debt and explain common causes of sovereign defaults.
- Describe the consequences of sovereign default.
- Describe factors that influence the level of sovereign default risk; explain and assess how rating agencies measure sovereign default risks.
- Describe characteristics of sovereign credit spreads and sovereign CDS and compare the use of sovereign spreads to credit ratings.

5.1 INTRODUCTION

Many large firms have business interests all over the world. Though these firms may be incorporated in a single country, it is important for them to assess the risks associated with the foreign countries they operate in. These risks are collectively referred to as country risk.

There are numerous components of country risk. One such component is political risk. For example, a change in government might lead to a less welcoming regulatory environment for foreign companies. In some extreme cases, there is the risk that a firm's assets in a foreign country will be seized after a political coup. Other risks may be concerned with the inadequacies of a country's legal system, the level of corruption, and the potential for violence.

Individuals and corporations can obtain diversification benefits by investing outside their domestic markets. One way an individual can do so is by investing in those firms in his or her country with interests throughout the world. However, a more direct way is through an investment in one or more funds specializing in certain countries or regions. However, the managers of these funds need to evaluate country risk when making their investments. An important question is: What return is required when investing in country X? Investments in country X may be riskier than investments in the domestic market, necessitating an extra expected return (risk premium).

Governments finance themselves by issuing bonds and money market instruments as well as by borrowing from large international banks. When lending to foreign governments, it is important for lenders to consider country risk as part of their credit default risk framework. To that end, the credit rating agencies we considered in the previous chapter provide ratings assessing the probability of a country defaulting. These agencies must consider whether the taxes a government collects will be sufficient to meet all its obligations, including those related to servicing outstanding debt.

We start this chapter by taking a big picture view of country risk. We look at several services that provide scores for countries by considering corruption, violence, legal risk, political risk, and overall risk. We then move on to explore the history of sovereign defaults as well as consider the performance and stability of sovereign credit ratings

5.2 EVALUATION OF RISK

Investors can add foreign investments to their portfolios more easily than ever before. For example, an investor in the United States can buy an emerging market exchange-traded fund (ETF) from Vanguard or Blackrock with a relatively low expense ratio. The fund manager will then invest in the equities of countries such as China, South Korea, Thailand, Peru, the Czech Republic, Malaysia, Turkey, Chile, Russia, Indonesia, Columbia, Poland, Namibia, Zambia, South Africa, Mexico, Brazil, Hungary, Morocco, and the Philippines.

Corporations have similar choices for the countries they can invest in. Many developing markets have economies that are growing faster than those of developed markets. However, this fast growth may be accompanied by higher economic risks and less stable political climates. Furthermore, there are often links between political and economic risks. If economic growth in a country slows, for example, dissatisfaction may lead to political turmoil and even lower future growth.

5.2.a GDP Growth Rates

The growth of a country's economy is measured its Gross Domestic Product (GDP). GDP is the total value of goods and services produced by all the people and firms in a country. Economists usually look at the growth rate in GDP after allowing for the impact of inflation (this is called the real GDP growth rate). For example, if the growth rate measured in domestic currency is 3% per year and inflation in the domestic currency is 2% per year, the real growth rate is only 1% (= 3% - 2%) per year.

An important consideration in assessing country risk is how a country will react to economic cycles. During economic downturns, for example, developing countries often see larger declines in GDP than their developed counterparts. This is because developing economies tend to rely more heavily on commodities. This means that they get squeezed by lower prices and demand during global recessions. For example, in 2009 the United States experienced its worst recession since the 1930s and saw a 2.8% decline in real GDP. At the same time, Mexico's real GDP shrunk by 4.7%. This was because Mexico's economy relied heavily on exports to its northern neighbour. One the other hand, some developing countries survived the 2007-2009 recession quite well. For example, China's real GDP growth rate was above 6% in 2009.

Table 5.1 gives the real GDP growth rates of several developing and developed countries in 2016 (compiled from the IMF World Economic Outlook database). The following table (source GARP curriculum) shows developing countries such as India, China, and the Philippines did well in 2016 and achieved real GDP growth rates of more than 6%.

Table 5.1 Positive and Negative Real GDP Growth Rates in 2016

Country	Real GDP Growth Rate (%)	Country	Real GDP Growth Rate (%)
Iceland	7.20	Russia	−0.25
Philippines	6.84	Nigeria	−1.54
India	6.83	Ecuador	−2.17
China	6.70	Argentina	−2.30
Vietnam	6.21	Brazil	−3.60
Kenya	6.00	Libya	−4.43
Ireland	5.22	Chad	−6.37
Indonesia	5.02	Yemen	−9.78

On the other hand, South Sudan and Venezuela had the worst GDP growth rates. These numbers illustrate some of the risks associated with developing countries. While South Sudan is rich in oil, its growth has been negatively impacted by a multi-year civil war. Venezuela is also rich in oil, but it has been held back by political turmoil and internal strife.

5.2.b Political Risk

Political risk is the risk that changes in governments, decisions made by governments, or the way governments operate will significantly affect the profitability of a business or an investment. Assessing political risk is never easy. Sometimes a change in leadership can transform the political landscape of a country and increase multiple forms of risk. As a result, investors tend to value government stability.

Whether democracies or authoritarian governments create more political risk is debatable. Authoritarian governments often do not change as frequently as democratically elected governments and can therefore pursue the same policies for a longer period. However, when there is a coup and one dictator takes over from another, there can be a sharp discontinuity in policy. In the case of democracies, governments may change relatively often, but the impact of the change is not usually as dramatic as it is with authoritarian governments.

There have been studies examining whether authoritarian governments lead to faster GDP growth than democratic governments. The results are mixed, with some studies arguing that countries with democratic governments grow faster and others arguing that countries with authoritarian governments grow faster. Some authors argue economic growth creates the desire for a democratic government, and so the causality may be the opposite of that assumed in other studies.

One aspect of political risk is corruption. When a business invests in a foreign country, it will almost certainly have to deal with the government bureaucracy. In some countries, as a practical matter, it may be impossible to do business without bribing public officials. This is a problem because bribery is illegal in many developed countries. For example, the Foreign Corrupt Practices Act (FCPA) is a statute in the United States prohibiting the bribing of foreign officials by Americans.

Bribes are an implicit tax on income that reduce profitability and returns for businesses operating in a country (and for investors in those businesses). The amount of money spent on bribes is typically uncertain (adding to the risk), and a firm may suffer (both financially and reputationally) if it is prosecuted in its home country.

Transparency International is an organization that uses surveys of experts living and working in different countries to compile a corruption index. The lower the index, the more corrupt the country is perceived to be. Table 5.2 shows the top 20 and bottom 20 countries in 2016.

Table 5.2 Corruption Indices Produced for the Top 20 and Bottom 20 Countries in 2016

Country	Corruption Index	Country	Corruption Index
Denmark	90	Cambodia	21
New Zealand	90	Uzbekistan	21
Finland	89	Burundi	20
Sweden	88	Cent. African Rep	20
Switzerland	86	Chad	20
Norway	85	Haiti	20
Singapore	84	Rep. of Congo	20
Netherlands	83	Angola	18
Canada	82	Eritrea	18

Another aspect of political risk is violence. Violence makes it difficult for businesses to operate, leads to higher insurance costs, and may lead to a difficult or totally unsatisfactory work environment for employees. An index measuring violence is the Global Peace Index. Table 5.3 shows scores for the best and worst countries in 2017. In this case, low scores are better than high ones.

Table 5.3 Global Peace Indices, 2017

Country	Peace Score	Country	Peace Score
Iceland	1.111	Israel	2.707
New Zealand	1.241	Palestine	2.774
Portugal	1.258	Columbia	2.777
Austria	1.265	Turkey	2.777
Denmark	1.337	Lebanon	2.782
Czech Republic	1.360	Nigeria	2.849
Slovenia	1.364	North Korea	2.967
Canada	1.371	Russia	3.047
Switzerland	1.373	Pakistan	3.058
Ireland	1.408	Rep. of Congo	3.061

Many countries rank similarly in Tables 5.2 and 5.3. As a result, doing business in countries such as Denmark, New Zealand, Switzerland, and Canada should lead to few of the corruption and violence problems we have mentioned. By contrast, many African and Middle Eastern countries are not as easy to operate in.

Another source of political risk is nationalization or expropriation. This is a particularly significant problem for firms working with natural resources. For example, a firm may own a mine or have the rights to drill for oil in a country. However, if the firm is profitable and the host country's economy is faltering, it can be a popular policy for a government to seize the foreign firm's assets while paying very little compensation. Of course, this may have the effect of discouraging future investment in the country, and thus hurting its economy in the long term.

5.2.c Legal Risk

Legal risk is the risk of losses due to inadequacies or biases in a country's legal system. A legal system that is trusted and perceived to be fair helps a country to attract foreign investment. Because business activities inevitably generate legal disputes, firms do not want to invest in a country where the legal system is biased, subject to government interference, and/or slow to the point of ineffectiveness. In this context, it is interesting to note that disputes between Russian oligarchs tend to be heard in British (rather than Russian) courts.

Property rights and contract enforcement are important aspects of a legal system. For example, if an investor is to buy shares issued by a firm domiciled in a foreign country, it is important for him or her to know that the country has fair and well-thought-out rules concerning the rights of shareholders and firm governance. Specifically, the country should have a legal system where a firm and its management can be sued in the event of insider trading, actions that hurt shareholders, or attempts to deceive the market about the firm's financial health.

The International Property Rights Index is published by the Property Rights Alliance to help individuals and firms understand the risks they are taking when they invest abroad. A country's total index is made up of a Legal and Political Index (which is subdivided into Rule of Law, Political Stability, and Control of Corruption), a Physical Property Index (which is subdivided into Property Rights, Registering Property, and Ease of Access to Loans), and an Intellectual Property Index (subdivided into Intellectual Property Protection, Patent Protection, and Copyright Protection). The overall index in 2017 varied from Yemen (2.73) to New Zealand (8.63). Table 5.4 gives examples of the indices that were compiled in 2017 for a few countries from different regions of the world. (Larger numbers indicate a better legal system.)

Table 5.4 International Property Rights Index for Sample Countries in 2017

Country	Overall Index	Legal and Political Index	Physical Property Index	Intellectual Property Index
Argentina	4.57	3.81	5.05	4.84
Australia	8.24	8.27	8.24	8.22
Brazil	5.43	4.44	6.12	5.75
Canada	8.18	8.37	7.91	8.26
China	5.71	4.52	7.00	5.61
Germany	7.96	7.84	7.66	8.38
Ghana	5.65	5.26	5.88	5.79

5.2.d The Economy

It is important to understand the economic risks associated with investing in a foreign country. GDP per capita and the real GDP growth rate tell part of the story, but it is also important to assess the country's competitive advantages and its level of economic diversification. For example, some countries are highly dependent on a single commodity. If the price of that commodity declines, the country and the value of its currency will suffer. Many African and Latin American countries fall into this category.

Countries, like firms, can develop competitive advantages. To quote Michael Porter, "[a] country's competitive advantage depends on the capacity of its industry to innovate and upgrade."3 Based on several years of research, Porter argues there are four key determinants of a country's competitive advantage.

1. Factor Conditions: The nation's position in factors of production (such as skilled labor or infrastructure) necessary to compete in an industry.
2. Demand Conditions: The nature of home-market demand for the industry's product or service.
3. Related and Supporting Industries: The presence (or absence) of supplier industries and other related industries that are internationally competitive.
4. Firm Strategy, Structure, and Rivalry: The conditions governing how firms are created, organized, and managed, as well as the nature of domestic rivalry.

Examples of countries developing competitive advantages are Hong Kong, Singapore, South Korea, and Taiwan (known collectively as the four Asian tigers). They maintained real GDP growth rates greater than 7% per year between the 1960s and 1990s. Hong Kong and Singapore have since become world-leading international financial centres, while South Korea and Taiwan are world leaders in manufacturing and information technology.

An important consideration when investing in a country is the extent to which its economy is diversified. Large countries such as Brazil, India, and China can broaden their economic bases without too much difficulty. Some small countries, however, rely on a small number of goods or services. This makes them very susceptible to changes in the demand for the goods and services they produce. In theory, countries should be able to hedge their risks by entering into long-term contracts with other countries for the sale of these goods (combined perhaps with long term contracts for the goods they need to import), but in practice this does not seem to happen to any great extent.

Another consideration here is a country may face a trade-off between short- and long-term growth. In the short-term, growth may be maximized by focusing on the extraction and export of a commodity. But for sustainable long-term growth, it might be preferable to develop other industries. In this respect, it is interesting to note that Saudi Arabia has an ambitious plan to restructure the Kingdom's oil-dependent economy by privatizing state assets and diversifying its focus.

5.3 TOTAL RISK

We have presented a few different components of country risk. However, it is natural to ask whether there exists a reliable composite risk measure (the equivalent of a VaR or expected shortfall for countries). There are some services that attempt to do this. One is Political Risk Services (PRS), which uses 22 measures of political, financial, and economic risk to calculate its index. Individual firms can customize the PRS forecasting model to their own projects or exposures by adjusting the weighting attached to each of the variables, adding or subtracting variables, or otherwise tailoring the model to emphasize specific potential sources of risk.

Media outlets, such as Euromoney and The Economist, also provide country risk scores. Euromoney bases its scores on a survey of 400 economists, whereas The Economist develops country risk scores internally based on currency risk, sovereign debt risk, and banking risk. The World Bank also provides country risk data measuring corruption, government effectiveness, political stability, regulatory quality, the rule of law, and accountability.

It is difficult to compare these services because they use different scoring methods and consider different attributes of country risk. It is also important to keep in mind that not every dimension of country risk is necessarily relevant to every individual or corporation. There is also the question of scaling. (If one country has a score twice as high as another, in what sense is it twice as risky?)

In many ways, the rankings of countries are more important than their numerical scores. Taken together, these services provide a useful narrative of risk in various countries. In some cases, the narrative accompanying a score is more important than either the score or the ranking itself.

5.4 SOVEREIGN CREDIT RISK

One measure of a country's risk is the risk it will default on its debt. There are two types of sovereign debt: the type issued in a foreign currency (such as the USD) and the type issued in the country's own currency. We will consider each in turn.

5.4.a Foreign Currency Defaults

Debt issued in a foreign currency is attractive to global banks and other international lenders. The risk for the issuing country, however, is that it cannot repay the debt by simply printing more money. To illustrate, the United States government can repay the debt it has issued in USD by printing more USD. This is referred to as increasing the money supply, and it may lead to inflation. A country such as Argentina, however, cannot do this.

There have been many defaults on sovereign debt over the last 200 years. Table 5.5 shows foreign currency sovereign defaults that happened between 2010 and 2016 (as reported by Moody's). Most of the defaults involved the exchange of old bonds for new bonds with some net present value loss to lenders. Greece was the biggest borrower to default during the 2010-2016 period. Moody's estimated the first 2012 Greek default resulted in investor losses of more than 70%, whereas the second default resulted in losses of more than 60%.

Table 5.5 Sovereign Foreign Currency Defaults 2010–2016

Country	Date	Debt Amount (Billions of USD)
Jamaica	Feb, 2010	7.9
Greece	Mar, 2012	261.5
Belize	Sep, 2012	0.5
Greece	Dec, 2012	42.1
Jamaica	Feb, 2013	9.1
Cyprus	Jul, 2013	1.3
Ukraine	Oct, 2015	13.3
Mozambique	Apr, 2016	0.7

Source: Moody's.

Defaults are caused by a combination of financial, economic, and political issues that were largely unforeseen at the times the loans were originally made. Many of the largest defaulters have been South American countries, and some have defaulted multiple times in the last 200 years. A default may make it impossible for a government to finance itself for a certain period. Over the longer term, however, debt markets have proved to be remarkably forgiving.

5.4.b Local Currency Defaults

Some countries have defaulted on debt issued in their own currency as well as on debt denominated in foreign currency. Two examples of this include the defaults of Brazil and Russia in 1990 and 1998 (respectively).

Research from Moody's indicates that countries are increasingly defaulting on both types of debt simultaneously. Why would countries default on debt denominated in their own currencies when they could simply print more money? There are several reasons.

- In the decades prior to 1971, currencies had to be backed by gold reserves. The amount of these reserves therefore limited a country's ability to print more money.
- Greece and other members of the European Union use the euro as their domestic currency. They do not, however, have the right to print euros (this is the responsibility of the European Central Bank). This means that Greece could not have solved its debt problems in 2012 by printing money.
- Printing more money debases the currency and leads to inflation. If firms in a country have foreign currency debt, debasing the value of the local currency (in which they earn their profits) can make it very difficult for them to repay loans. This would have negative consequences for the local economy.

On the other hand, printing money is likely to be attractive in the short term because a country's reputation and credit rating will not immediately suffer.

Rating agencies, as we will see, typically provide both local currency ratings and foreign currency ratings. The local currency rating for a country is an opinion about the possibility of a country defaulting on its local currency debt. The foreign currency rating is an opinion about the possibility of it defaulting on its foreign currency debt. The local currency rating is typically one or two notches higher than the foreign currency rating.

5.4.c Impact of a Default

When a firm defaults, its creditors usually have the right to force it to liquidate. While they may end up getting less than the face value of the debt, at least the situation is resolved. When a country defaults, however, it cannot be liquidated. Usually, the old debt is replaced by new debt or is restructured in some other way (e.g., by lowering the principal, lowering the interest payments, or extending the life of the debt). In the past, a default on debt might have been followed by military action (such as what happened to Venezuela in the 1900s). However, this does not happen in the modern era.

The modern consequences of a default by a sovereign nation include:

- A loss of reputation along with an increased difficulty in raising funds for several years,
- A lack of investors willing to buy the debt and equity of corporations based in the country,
- An economic downturn, and
- Political instability as the population loses faith in its leaders.

Researchers have also found that a default negatively affects Table 5.6 Debt as Percent of GDP in 2017 GDP growth, negatively affects the country's credit rating for many years, can hurt exports, and can make the defaulting country's banking systems more fragile.

Table 5.6 Debt as Percent of GDP in 2017

Country	Government Debt (% of GDP)
Japan	240.30
United States	108.14
France	96.84
United Kingdom	89.48
Brazil	83.36
India	68.69
Germany	65.01
China	47.61
Russia	17.35

In short, defaulting on debt is not something countries should take lightly. It can seriously impede their economic development and growth. Often, the International Monetary Fund becomes involved in restructuring the debt and imposing strict austerity conditions on the defaulting country. For example, it can insist government budget deficits are reduced through spending cuts, tax increases, or a combination of both.

Typically, defaults happen only after a country has experienced very difficult economic conditions or political upheavals. For example, a large default by Argentina in 2001 was a result of an economic depression starting in the third quarter of 1998. This caused widespread unemployment, riots, and the fall of a government.

5.5 SOVEREIGN CREDIT RATINGS

Rating agencies look at several factors when rating countries. It is obviously important to consider the amount of debt a country already has. In the case of a firm, the ratio of the book value of debt to the book value of equity is a well-known leverage ratio. Because there is no equity measure for countries, however, rating agencies instead look to the ratio of government debt to GDP.

Table shows statistics on the debt-to-GDP ratio for large developed and developing countries in 2017. Japan appears to be an outlier in these statistics, but there is a reason for this. The Japanese government, to a much greater extent than other governments, holds assets. These assets include cash, securities, and real estate holdings. The Japanese government also holds some of its own bonds.9 When the figures are adjusted for those asset holdings not earmarked for other purposes (e.g., pension payments), the debt-to-GDP ratio becomes much more reasonable. Indeed, depending on exactly how the adjustments are made, the debt-to-GDP ratio becomes somewhere in the 40% to 90% range.

It can be relevant to look at how debt ratios have changed over time. In the United States, the debt-to-GDP ratio was in the 30% to 40% range between 1966 and 1984. It was in the 40% to 60% range for most of the period between 1985 and 2005. By 2017, the ratio had risen to over 108%.

It is worth noting debt issued by a government is not a country's total indebtedness. In the United States, for example, states such as California, and municipalities such as New York City also borrow money. It is also useful to distinguish between debt held internally (by a country's own citizens, banks, or other corporations) and that held externally (by foreign investors).

There are several other factors that are considered when determining a rating.

- Social Security Commitments: Governments make commitments to their citizens to pay pensions and provide health care. As the size of these commitments increases, a government has less free cash to service debt.
- The Tax Base: A rating agency must assess the size and reliability of the tax base. Countries with diversified economies will tend to have a more stable tax base than those that depend on one or two industries.
- Political Risk: It is sometimes argued autocracies are more likely to default than democracies. Given that the alternative to default is printing money, the decision-making process at the country's central bank may be important. Thus, it is important to know the extent to which the central bank is independent of the government.
- Implicit Guarantees: Countries in the Eurozone may well be helped by rich member countries (e.g., Germany and France) when they get into financial difficulties. However, there is no explicit guarantee that this help will always be given.

Table compares the default rate experienced by sovereign countries with the default rate experienced by corporations with the same rating for periods ending in 2016 (the data for corporations was considered in Chapter 4). Countries rated AAA and AA have performed very well over the following ten-year period, whereas countries with an A rating have not performed as well. Sovereign foreign debt with BBB, BB, and B ratings has performed about the same as corporate debt with those ratings, but sovereign local debt has performed better than corporate debt. A CCC to C rating for sovereign foreign debt has a much higher ten-year default probability than the same rating for corporate debt, but local debt with that rating has a lower ten-year default probability.

As mentioned earlier, a country's local debt is typically rated slightly higher than its foreign debt. However, debt denominated in local currency has performed better than debt denominated in foreign currency in most categories. The ten-year default rate on the former is about half of that on the latter.

Table of Chapter 4 shows one-year transition rates for corporations. In Table 5.8, we show a table of rating transitions for the foreign debt of countries. In Table 5.9, we do the same for local debt. The tables show the rating transitions of countries are more stable than those of corporations except for the lowest rating category. The probability that corporations rated AAA, AA, A, BBB, BB, and B will keep that rating for one year are 87.05%, 86.82%, 87.79%, 85.56%, 76.98%, and 74.26% (respectively). Tables show the corresponding numbers are higher for both sovereign local debt and sovereign foreign debt.

Table 5.7 Comparison of Sovereign Foreign Currency and Sovereign Debt Denominated in the Local Currency

	Ten-Year Cumulative Default Rate		
Rating	Corporations	Sovereign Foreign Debt	Sovereign Local Debt
AAA	0.72	0.00	0.00
AA	0.77	0.00	0.10
A	1.41	5.80	5.80
BBB	3.76	3.60	2.90
BB	13.33	8.60	3.20
B	25.43	26.60	10.00
CCC/C	51.03	79.30	43.70
Investment Grade	2.11	2.00	2.20
Speculative Grade	21.67	19.40	8.10
All Rated	9.18	7.70	4.00

Source: Standard & Poor's.

Table 5.8 One-Year Rating Transitions for Debt Denominated in a Foreign Currency from 1975–2016. NR Indicates a Transition to the Not-Rated Category

	AAA	AA	A	BBB	BB	B	CCC/CC	Default	NR
AAA	96.7	3.2	0.0	0.0	0.1	0.0	0.0	0.0	0.0
AA	2.8	93.2	2.7	0.4	0.3	0.0	0.0	0.0	0.6
A	0.0	3.9	90.5	4.9	0.4	0.0	0.0	0.0	0.3
BBB	0.0	0.0	5.2	89.6	4.5	0.5	0.2	0.0	0.0
BB	0.0	0.0	0.0	5.8	86.9	5.9	0.7	0.5	0.2
B	0.0	0.0	0.0	0.0	5.9	87.4	3.0	2.5	1.2
CCC/CC	0.0	0.0	0.0	0.0	0.0	32.9	28.9	38.2	0.0

Source: Standard & Poor's.

Table 5.9 One-Rating Transitions for Debt Denominated in the Local Currency from 1993–2016. NR Indicates a Transition to the Not-Rated Category

	AAA	AA	A	BBB	BB	B	CCC/CC	Default	NR
AAA	95.8	4.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AA	1.7	90.7	5.8	1.1	0.0	0.0	0.0	0.0	0.7
A	0.0	2.7	90.2	6.2	0.4	0.2	0.0	0.0	0.2
BBB	0.0	0.0	5.2	87.3	6.6	0.7	0.1	0.0	0.0
BB	0.0	0.0	0.0	4.8	84.8	8.2	1.1	0.8	0.2
B	0.0	0.0	0.0	0.0	6.6	87.4	2.6	1.7	1.7
CCC/CC	0.0	0.0	0.0	0.0	0.0	52.5	37.1	10.4	0.0

Source: Standard & Poor's.

5.6 5CREDIT SPREADS

The credit spread for sovereign debt in a specific currency is the excess interest paid over the risk-free rate in that currency. There is a strong correlation between credit spreads and ratings, but credit spreads can provide extra information on the ability of a country to repay its debt. One reason for this is that credit spreads are more granular than ratings (i.e., there is a range of credit spreads associated with a credit rating). One country with a given rating might have a lower credit spread (and therefore be perceived as less risky) than another country with the same rating.

Credit spreads also have the advantage of being able to adjust more quickly to new information than ratings. However, they are also more volatile. Recall from Chapter 4 that rating stability is one of the objectives of rating agencies. This is true for country bond ratings as well as for corporate bond ratings. As mentioned in Chapter 4, a rating is changed only when there is reason to believe the long-term financial health of the firm or country has changed.

One source of credit spread data is the credit default swap market. Credit default swaps are traded on countries as well as corporations. They are like insurance contracts in that they provide a payoff to the holder if the country defaults within a certain period (usually five years). Roughly speaking, the payoff is designed to put the bondholder in the same position as he or she would have been if the bond had not defaulted. Unlike an insurance contract, a CDS can be used by speculators as well as hedgers. For example, an entity does not have to have exposure to Brazilian debt to buy protection against a Brazilian default. If Brazil does not default, the speculator pays a premium and receives no payoff. If Brazil does default, the speculator receives the corresponding payoff even if he or she does not have any real exposure.

There has been some controversy about the actions of speculators in the sovereign credit default swap market. Speculators were blamed by some for driving up Greek CDS spreads in 2010. It was claimed this also drove up the credit spread for bonds issued by Greece, making the country's financial problems more severe.

To see how this could happen, suppose the euro risk-free rate is 3% and the yield on Greek debt is 10%. If speculators have driven up the cost of protection on the debt to 9%, bond holders will consider a 10% yield to be too low because the combined (close-to-risk-free) net return would be only 1% (= 10% — 9%). This is less than the 3% they could earn elsewhere.

The European Union became concerned about the potential for speculators to influence the bond market in the way we have just described. As a result, it introduced legislation banning the purchase of uncovered sovereign CDS contracts (i.e., CDS contracts where the purchaser does not own bonds issued by the country and has no exposure to a default by the country). However, it is important to note most researchers argue Greek bond yields were not affected by CDS spreads.

The bonds issued by a country may, in practice, be a better source of credit spread data than the CDS market. In a CDS, there is always the possibility the seller of protection will default (these concerns rose during the 2007-2008 credit crisis). The CDS market is also subject to illiquidity problems as well as clustering (i.e., where the CDS spreads of groups of countries move together in a way that does not necessarily reflect default risk).

Reading 06 Measuring Credit Risk

Learning Objective

- Evaluate a bank's economic capital relative to its level of credit risk.
- Explain the distinctions between economic capital and regulatory capital, and describe how economic capital is derived. Identify and describe important factors used to calculate economic capital for credit risk: probability of default, exposure, and loss rate.
- Define and calculate expected loss (EL).
- Define and explain unexpected loss (UL).
- Estimate the mean and standard deviation of credit losses assuming a binomial distribution.
- Describe the Gaussian copula model and its application.
- Describe and apply the Vasicek model to estimate default rate and credit risk capital for a bank.
- Describe the CreditMetrics model and explain how it is applied in estimating economic capital.
- Describe and use the Euler's theorem to determine the contribution of a loan to the overall risk of a portfolio.
- Explain why it is more difficult to calculate credit risk capital for derivatives than for loans.
- Describe challenges to quantifying credit risk.

6.1 INTRODUCTION

In this reading we will discuss the credit risk by considering risk in the loan portfolio of banks. We learned in Book 3 Reading Banks, the banks need to maintain equity capital to absorb losses on loan defaults of borrower. The equity capital should be enough to absorb these losses. The capital maintained to absorb losses as per own estimates is economic capital. In credit risk concept, in this reading we will discuss the calculation / measurement of credit risk which is the basis of economic capital.

Note: This reading requires understanding of lots of basic concepts which are not discussed in FRM Part I. The basics are covered in FRM Part II Subject Credit Risk and these concepts are also discussed in same subject. Hence in this reading we will stick to exam related discussion and will avoid touching detailed discussion which might be confusing without knowing basics. We don't have to worry about mission certain aspect of concepts because same concepts are repeated in Part II and we can get the detailed understanding in Part II.

We consider three different models for quantifying credit risk in this chapter.

- First model considers mean and standard deviation of the loss from loan portfolio is determined from the properties of the individual loans.
- The second model, known as the Vasicek model, is used by banks regulators to estimate an extreme percentile of the loss distribution which determines the banks regulatory capital.
- The third model is known as CreditMetrics and is often used by banks themselves when estimating economic capital.

Background

As we discussed capital should be enough to absorb losses. Assume bank has the capital of $5 billion. If the bank incurs the loss of $1 billion, capital is sufficient to keep bank going concern. Bank should maintain sufficient capital to absorb sufficient loss even in extreme scenarios. Assume there is 1% chance of loss exceeding capital maintained by bank, this means bank is likely to fail once in 100 years.

Banks maintain debt capital and equity capital. The debt capital is subordinate to deposits. This means if banks equity capital is wiped out by losses, the debt holder should incur losses before depositors are impacted.

Equity capital is called as going concern because bank is solvent until its wiped out. Banks debt capital is called as gone concern capital because it only absorb loss when bank is already failed.

The Basel Committee

For the first time Basel I provided common approach to determine the required credit risk capital for banks under supervision. In 1996 the Basel accord was extended by requiring capital for market risk as well as credit risk. In 1999, Basel II was proposed introducing capital requirement for operational risk and changed capital requirement for credit risk.

Basel II credit risk regulation, which still underlie credit risk capital calculation, feature a standardized approach and internal rating based approach IRB,.

Economic capital

In economic capital, correlations between the risk are often considered however, for regulatory capital correlations are ignored. In this reading we will discuss CreditMetrics model for calculating credit risk economic capital. This model considers the losses from both credit rating downgrade and loan defaults.

6.2 MEASURING CREDIT RISK FOR BANKS

Before we proceed further, to actual discussion and you get lost in the complexity of these topics, lets first discuss what is the purpose of the following discussion.

Basel II capital requirement for banks that use the IRB approach is given by

$$(WCDR - PD) X\ LGD\ X EAD$$

WCDR is worst case default rate or 99.9 percentile default rate.

PD is probability of default, LGD is loss given default and EAD is loan amount called as exposure at default.

In the above equation, Probability of default PD and Loss given default LGD are used. We will learn the calculation of PD and LGD in FRM Part II and in all the following discussions we will assume some values for PD and LGD. So do not bother about the source of these values because these are assumed. EAD is exposure at default which is known and specific for the bank and only banks can calculate it so this value is also assumed.

This paragraph is written to reduce your fear of what's coming next and explains what is provided in following section and is not the part of GARPs discussion. So, feel free to skip this para if you want. The following discussion manly focuses on WCDR. WCDR is the default rate at 99.9 percentile. We can calculate 99.9% default rate using Vasicek model and this is the reason why we are discussing this model. Because Vasicek model takes the support of one factor model which gives us Ui hence we will talk about the one factor model. The one factor model takes the support of Gaussian Capula hence we will learn about this Capula. Capula assumes mean and standard deviations and hence the same discussion. So all the discussion below is to reach at WCDR calculation. Some of you might feel intimidated by this discussion but do not worry about the complexity part because you don't need to understand everything in very detail. Even if you just know what's written is mostly sufficient for exam. Secondly, while working in organizations, unless you are working as hardcore quants modeler, you will never see these model's structure. All these models are already coded in algorithm (software) and our job is to feed the inputs to these models and will get the final result. Our job is to interpret the output provided by these models.

6.3 EXPECTED AND UNEXPECTED LOSS

Expected loss is the loss on portfolio which is expected and this loss is charged in interest rates charged by banks to borrowers. Hence, expected loss is rarely a cause of concern for banks. Expected loss can be calculated as

$$EL = EAD\ X\ PD\ X\ LGD$$

Where, EAD is exposure at default (loan amount)

PD is probability of default and LGD is loss given default.

The major concern for the bank is unexpected loss which is the difference between actual loss and loss expected by bank. The following discussion cover the measurement process of unexpected loss.

Binomial Distribution for defaults

For the defaults among the borrowers we expect the default rate would be similar if the borrowers are independent of each other. Consider, 1000 loans in banks lending portfolio has 1% probability of default within a year, this means there will be 10 defaults every year. However, this assumption of independence of loan default is not plausible. The banks face good times and bad times in reference to loan default. An important reason why companies do not default independently of each other is the economy. Lean defaults under the assumption of independent defaults follow binomial distribution. A lognormal distribution can also be used to provide approximate fit to default data.

6.4 THE MEAN AND STANDARD DEVIATION OF CREDIT LOSSES

Suppose a bank has n loans and lets assume the following quantities

L_I = the loan amount in ith loan (assumed constant throughout year)

P_i = Probability of default for the ith loan

R_i = The recovery rate in the event of default by the ith loan

ρ = the correlation between loss on the ith and jth loan

σ_i = the standard deviation of los from the ith loan

σ_p = he standard deviation of loss from the portfolio

α The standard deviation of portfolio loss as a fraction of the size of the portfolio

If the loan defaults, the loss is Loan amount X Loss rate i.e. Loan amount (1-Recovery rate) (L(1-R)). The probability distribution for the loss from ith loan consist of probability $P_i$that there will be loss of this amount and a probability (1-p) that there will be no loss. This is a binomial distribution.

The mean loss is = $E(Loss) = p_i L_i (1 - R_i)$

Standard deviation = $\sigma = \sqrt{p - p^2}$[L(1-R)]

The standard deviation is the simplified formula by assuming all the loans have the same principle amount L all the recovery rates are the same and equal to R, all default probabilities are the same and equal to p. This gives us $\sigma_i = \sigma$

The standard deviation of the loss from the loan portfolio as a percentage of its size,

$$\alpha = \frac{\sigma_P}{nL} = \frac{\sigma\sqrt{1 + (n-1)\rho}}{L\sqrt{n}}$$

Illustration

Bank has 100,000 loans and each loan is USD 1 million and has 1% probability of default in a year. The recovery rate is 40%. In this case to calculate the standard deviation and the α

N= 100,000, p = 0.01, R = 0.40 and L = 1 so that

$$\sigma = \sqrt{(0.01 - 0.0001)} X\ 1\ X\ 0.6 = 0.597$$

$$\frac{0.0597\ X\ \sqrt{1 + (99999 X\ 0.1)}}{\sqrt{100000 X\ 1}} = 0.0189$$

This gives us the relationship between α *and* ρ. As we can see from the formula, α increases as ρ increases. The rate of increase is greater for smaller values of ρ.

To calculate bank capital using above model, we need to estimate correlations (using historical data) and then estimate the unexpected loss. Also rating agencies estimate the correlation between the default rates of two companies as a function of their rating.

6.5 THE GAUSSIAN CAPULA MODEL

Assume the probability distribution for variables V1 and V2 are known and we want to define the complete way in which they depend on each other in the form of joint probability distribution. First we assume the bivariate normal distribution as long as both the variables are normally distributed. In case of non normal distributions we can transform these distributions into standard normal distribution by mapping percentile of original distribution to standard normal distribution. After transformation V1 is U1 (normal) and V2 is U2 (normal) with mean zero and standard deviation 1.

We then assume the distribution of U1 and U2 are bivariate normal with particular correlation. This gives us the joint distribution of U1 and U2. The transformation is very important because it is not possible to define the joint distribution of V1 and V2 because the unique nature of distribution.

One factor correlation model

When there are many different distributions which is common in loan portfolios, this involves specifying a large number of different correlation parameters. This problem can be handled by one factor model

$$U_i = a_i F + \sqrt{1 - a_i^2 Z_i}$$

Where, F is the common factor to all the Ui, Zi is the component of Ui unrelated to the common factor F. The ai parameters with values between -1 to +1. The variable F and Zi have standard normal distribution and Variable Ui is the sum of two independent normal distributions and therefore normal. It has mean zero because the components are mean zero. The standard deviation of Ui is 1.

The final result of this process (Check GARP for detailed process), the coefficient of correlation between Ui and Uj becomes

$$E(a_i a_j F^2)$$

In the next section the one factor model described above will be applied to default probabilities. The factor can be thought of as a variable related to the economy that affects default rates.

6.6 THE VASICEK MODEL

The Vasicek model is used by regulators to determine capital for loan portfolios. It uses the Gaussian copula model to define the correlation between defaults. The Vasicek model is better compared to previous models because the unexpected loss can be determined analytically.

Assume the probability of default PD is the same for all companies in large portfolio. The binary probability of the default distribution for company I for one year is mapped to the standard normal distribution Ui. Value in extreme left tail of this standard normal distribution correspond to a default whereas the rest of the distribution corresponds to no default. This gives us

The Company i defaults if $Ui \leq N^{-1}(PD)$,

The term $N^{-1}(PD)$ is the inverse cumulative distribution which simply gives us the z value for the probability.

So for the PD of 1%, $N^{-1}(0.01) = -2.326$. This means values between left infinity to -2.326 corresponds to default.

To simplify model we assume ai are same for all *i.* hence model is

$$U_i = aF + \sqrt{1 - a^2 Z_i}$$

F is the factor represents the recent health of the economy. Higher F means economy is doing well and all Ui will tend to be high i.e. defaults are unlikely. Reverse is true when F is low. For each value of F the distribution of each Ui has a mean of aF and a standard deviation of $\sqrt{1 - a^2}$. For a large portfolios, the default rate is the probability Ui is less than $N^{-1}(PD)$.

Default rate (as a function of F) = $N\left(\frac{N^{-1}(PD) - aF}{\sqrt{1-a^2}}\right)$

Assuming the 0.1% default probability and each correlation pair of Ui distributed as $\rho = a^2$, the formula is converted into

99.9 percentile for default rate = $N\left(\frac{N^{-1}(PD) + \sqrt{\rho}N^{-1}(0.0001)}{\sqrt{1-\rho}}\right)$

The probability of default of 0.01% gives the a portfolio default rate which converts into one in every 1000 year default.

Illustration

Consider PD = 1.5% and $\rho = 0.4$. The 99.9 percentile for the portfolio default rate is

$N\left(\frac{N^{-1}(0.015) + \sqrt{0.4}N^{-1}(0.0001)}{\sqrt{1-0.4}}\right)$ = 39.0%

6.8 CREDITMETRICS

CreditMetrics is the model banks often use to determine economic capital. Under this model, each borrower is assigned an external or internal credit rating. A one-year transition table (like that discussed in Chapter 4) is used to define how ratings change.

The bank's portfolio of loans is valued at the beginning of a one-year period. A Monte Carlo simulation is then carried out to model how ratings change during the year. In each simulation trial, the ratings of all borrowers at the end of the year are determined, and the portfolio is revalued. The credit loss is calculated as the value of the portfolio at the beginning of the year minus the value of the portfolio at the end of the year. The results of many simulation trials are used to produce a complete credit loss distribution.

As we have stressed throughout this chapter, the bank's borrowers generally do not default independently of each other. To reflect this, we need to sample from the distributions in a way that builds in a correlation between the samples. A factor model is usually used to define correlations between the normal distributions. The Monte Carlo simulation is therefore an implementation of the Gaussian copula model. The probability distribution of rating transitions for each borrower is transformed into a normal distribution, and the correlations are those between the normal distributions (not those between the rating transitions themselves). The correlations between the returns on traded equities are often used to define the correlations used in CreditMetrics. This is an approach that can be justified using Merton's model(discussed in FRM Part II). Recall from Chapter 4 that this is a model where a company defaults when the market value of its assets falls below the book value of its debt.

How CreditRisk Metrics is different from Vasicek? (Remember for Exam)

The limitation of Vasicek is that it does not consider the impact of rating changes, however, RiskMetrics considers the impact of default and rating downgrade both on credit loss.

6.9 RISK ALLOCATION

A result developed by a famous mathematician, Leonhard Euler, many years ago can be used to divide many of the risk measures used by risk managers into their component parts. Euler's result is concerned with what are termed homogeneous functions. Many risk measures are homogeneous functions. Indeed, we learned in Chapter 1 that homogeneity is one of the properties of a coherent risk measure. If a portfolio is changed so that each position is multiplied by some constant λ, a risk measure is usually multiplied by λ. Euler's theorem therefore gives us a way of allocating a risk measure F that is a function of many different trades into its component parts.

Illustration

Suppose the losses from loan A B and C have standard deviation of 1.1, 0.9 and 0.9.

The correlation between the losses Loan A to B and C equals zero, Loan B to Loan C is 0.7.

The standard deviation of total loss is

$$\sqrt{1.1^2 + 0.9^2 + 0.9^2 + 2\,x\,0.7\,x\,0.9\,x\,0.9} = 1.99\%$$

Now if the size of loan increased to 1.111 (i.e. 1% change), the increase in standard deviation of loan portfolio is

$$\sqrt{1.1^2 + 0.9^2 + 0.9^2 + 2\,x\,0.7\,x\,0.9\,x\,0.9} - \sqrt{1.111^2 + 0.9^2 + 0.9^2 + 2\,x\,0.7\,x\,0.9\,x\,0.9} = 0.006098$$

As per Euler's Theorem, the relationship between the functions and variables can be given by,

$$Q_i = X_i \frac{Snall\ change\ in\ variable\ X}{Resultant\ cahnge\ in\ variable\ F}$$

Euler's provided that as X gets smaller, Risk function F is the sum of Q components

$$F = \sum Q_i$$

We can calculate the Q1 component as 0.006098/0.01 = 0.7733

Simlarly by increasing each loan by 1% at a time, we get

For loan 2: Q2 = 0.6924

For loan 3: Q3 = 0.6924

Q1 + Q2 + Q3 = 0.6098 + 0.6924 + 0.6924 = 1.99.

In this way the portfolio standard deviation of loss can be decomposed into individual contribution of loss.

Reading 07 Operational Risk

Learning Objectives

- Describe the different categories of operational risk and explain how each type of risk can arise.
- Compare the basic indicator approach, the standardized approach, and the advanced measurement approach for calculating operational risk regulatory capital.
- Describe the standardized measurement approach and explain the reasons for its introduction by the Basel committee.
- Explain how a loss distribution is derived from an appropriate loss frequency distribution and loss severity distribution using Monte Carlo simulations.
- Describe the common data issues that can introduce inaccuracies and biases in the estimation of loss frequency and severity distributions.
- Describe how to use scenario analysis in instances when data is scarce.
- Describe how to identify causal relationships and how to use Risk and Control Self-Assessment (RCSA), Key Risk Indicators (KRIs), and education to measure and manage operational risks.
- Describe the allocation of operational risk capital to business units.
- Explain how to use the power law to measure operational risk.
- Explain the risks of moral hazard and adverse selection when using insurance to mitigate operational risks.

7.1 INTRODUCTION

Understanding operational risk has become increasingly important for banks, insurance companies, and other financial institutions. There are many ways operational risk can be defined. It is sometimes defined very broadly as any risk that is not a market risk or a credit risk. A much narrower definition would be that it consists of risks arising from operational mistakes; this would include the risk that a bank transaction is processed incorrectly, but it would not include the risk of fraud, cyberattacks, or damage to physical assets.

Operational risk has been defined by the Basel Committee as:

The risk of loss resulting from inadequate or failed internal processes, people, and systems or from external events.

The International Association of Insurance Supervisors defines operational risk similarly as:

The risk of adverse change in the value of capital resources resulting from operational events such as inadequacy or failure of internal systems, personnel, procedures, or controls, as well as external events.

These definitions include the risks arising from computer hacking, fines from regulatory agencies, litigation, rogue traders, terrorism, systems failures, and so on. However, they do not include strategic risks or reputational risks.

Seven categories of operational risk have been identified by the Basel Committee.

1. Internal fraud: Acts of a type intended to defraud, misappropriate property, or circumvent regulations, the law, or company policy (excluding those concerned with diversity or discrimination) involving at least one internal party. Examples include intentional misreporting of positions, employee theft, and insider trading on an employee's own account.

2. External fraud: Acts by a third party of a type intended to defraud, misappropriate property, or circumvent the law. Examples include robbery, forgery, check kiting, and damage from computer hacking.

3. Employment practices and workplace safety: Acts inconsistent with employment, health or safety laws or agreements, or which result in payment of personal injury claims, or claims relating to diversity or discrimination issues. Examples include workers compensation claims, violation of employee health and safety rules, organized labor activities, discrimination claims, and general liability (for example, a customer slipping and falling at a branch office).

4. Clients, products, and business practices: Unintentional or negligent failure to meet a professional obligation to clients and the use of inappropriate products or business practices. Examples are fiduciary breaches, misuse of confidential customer information, improper trading activities on the bank's account, money laundering, and the sale of unauthorized products.

5. Damage to physical assets: Loss or damage to physical assets from natural disasters or other events. Examples include terrorism, vandalism, earthquakes, fires, and floods.

6. Business disruption and system failures: Disruption of business or system failures. Examples include hardware and software failures, telecommunication problems, and utility outages.

7. Execution, delivery, and process management: Failed transaction processing or process management, and disputes with trade counterparties and vendors. Examples include data entry

errors, collateral management failures, incomplete legal documentation, unapproved access given to client's accounts, non-client counterparty mis performance, and vendor disputes.

Operational risk is much more difficult to quantify than market or credit risk. In the case of market risk, risk factor volatilities can be estimated from historical data so that risk measures (such as VaR) can be used to quantify potential losses. In the case of credit risk, rating agencies and internal bank records can provide data on expected and unexpected losses. For operational risk losses, however, there is relatively little data. What is the probability of a cyberattack destroying bank records? What is the probability of a rogue trader loss? How much would the bank lose if these events occurred? Many serious operational risks facing financial institutions involve rare and novel events.

This chapter looks at the way regulatory and economic capital is calculated for operational risk. It also looks at how financial institutions can take proactive steps to both reduce the chance of adverse events happening as well as minimize the severity of the outcomes when they do happen.

7.2 LARGE RISKS

We start this chapter by reviewing three large operational risks faced by financial institutions.

7.2.a Cyber Risks

Cyber risk is one of the largest risks faced by financial institutions. They and their clients have benefited from the development of credit and debit cards, online banking, mobile wallets, electronic funds transfer, and so on. However, these innovations have also created opportunities for hackers. Cyberattacks are proving to be very expensive for businesses. These attacks are increasing in sophistication and severity with each passing year. Threats can come in the form of individual hackers, nation states, organized crime, and even insiders. Defenses developed against cyber threats include user account controls, cryptography, intruder detection software, and firewalls.

Cyber-crime includes the destruction of data, theft of money, theft of intellectual property, theft of personal and financial data, embezzlement, fraud, and so on. For example, a 2013 cyberattack at Yahoo saw a data breach that included the names, email addresses, telephone numbers, security questions and answers, dates of birth, and passwords for three billion user accounts. Equifax, a large consumer credit reporting agency, reported in cyberattack in 2017 affecting 143 million people in the United States. Sensitive information such as social security numbers and driver license numbers were obtained by the hackers.

Large corporations are under continuous attack by cyber criminals. Most attacks are unsuccessful, but the number of reported successful attacks appears to be rising each year. In addition, there may be many successful cyberattacks that go unreported.

Financial institutions are targeted in many ways. Most readers will be familiar with a common hacking practice called phishing. While phishing can come in many forms, a common situation involves a hacker targeting a financial institution's customers with an email asking them to confirm account information. If a customer complies, the criminal gains access to the customer's account. Sometimes the customer is tricked into installing malicious software that allows the hacker to capture sensitive information as it is typed.

A more serious threat is to the financial institution itself. If a hacker can gain access to a financial institution's systems, he or she can obtain client information, delete records, enter false transactions, embezzle funds, and so on. The dangers here were illustrated in March 2016 hacking of the Central Bank of Bangladesh. The hackers found multiple entry points into the bank's network and planned to embezzle over USD 1 billion through a series of international transactions. However, it is reported that because of a data entry mistake, they obtained only USD 80 million (which is still an embarrassingly large sum).

All companies should accept that, however good their defenses are, they are liable to be hacked in the next few years. They should have plans that can be implemented at short notice to deal with attacks of different severities. In some instances, an extreme response to an attack, such as delaying the acceptance of new transactions for a few days, might be necessary.

7.2.b Compliance Risks

Compliance risk is another operational risk facing financial institutions. This is the risk that an organization will incur fines or other penalties because it knowingly or unknowingly fails to act in accordance with industry laws and regulations, internal policies, or prescribed best practices. Activities such as money laundering, terrorism financing, and assisting clients with tax evasion can all lead to big penalties.

A well-known example of compliance risk is Volkswagen's failure to comply with U.S. emissions standards by cheating during emissions testing. This led to a fine of about USD 2.8 billion.

One example of compliance risk in the financial sector is the USD 1.9 billion penalty paid by HSBC in 2012. In the years preceding the fine, the bank did not implement anti-money laundering programs for its Mexican branches. As a result, Mexican drug traffickers were able to illegally deposit large sums of money in cash. HSBC eventually reached a deferred prosecution agreement with the United States Department of Justice, which required it to pay a large fine and retain an independent compliance monitor.

Another example of financial sector compliance risk comes from French bank BNP Paribas. In 2014, it was announced the bank would pay USD 8.9 billion (roughly one-year's profit) to the United States government for moving dollar-denominated transactions through the U.S. banking system on behalf of Sudanese, Iranian, and Cuban parties. These transactions occurred despite the fact all three countries were subject to sanctions by the U.S. government. In addition to paying the fine, BNP Paribas was also banned from conducting certain U.S. transactions for a year.

Regulatory infractions can result from a small part of a large company's global activities. However, they can be very expensive (both in terms of fines and loss of reputation). It is important for financial institutions to have systems in place to ensure that they are following all applicable laws and regulations. In this regard, technology can be of help. For example, some banks have developed systems designed to detect suspicious requests to open accounts or transfer funds in real time.

7.2.c Rogue Trader Risk

Rogue trader risk is the risk that an employee will take unauthorized actions resulting in large losses. One of the most notorious incidents involved Barings Bank trader Nick Leeson. His job was to do relatively low-risk trades from the firm's Singapore office. Due to flaws in the Barings systems, however, he found a way of taking large risks and hiding losses in a secret account. His attempts to recoup losses led to even more losses (which exceeded USD 1 billion) and he was forced to flee Singapore to avoid prosecution (leaving a note saying he was sorry). Leeson was eventually returned to Singapore, prosecuted, and received a prison sentence. Barings Bank, which had been in existence for 200 years, was forced into bankruptcy.

Another large loss occurred at the Societe Generale (SocGen). Ostensibly, trader Jerome Kerviel was tasked with finding arbitrage opportunities in equity indices, such as the German DAX, the French CAC 40, and the Euro Stoxx 50. These might arise if a futures contract on an index was trading at different prices on two exchanges, or at a price inconsistent with the prices of the underlying shares. However, Kerviel found a way of speculating while appearing to arbitrage. He took big positions and created fictitious trades to make it appear as if he was hedged. In January 2008, Kerviel's unauthorized trading was uncovered and SocGen lost EUR 4.9 billion when his positions were closed out.

Other rogue trading incidents include a USD 2.3 billion loss at UBS in 2011 and a USD 700 million loss at Allied Irish Bank in 2002. The common theme among these losses is that a single trader working for a bank was able to take huge risks without the firm's knowledge or authorization. These actions were hidden via the creation of fictitious offsetting trades, or in some other way. In every case, the trader hoped that by continuing to speculate, losses would have been offset, and the unauthorized trading would have then have been forgiven. (Indeed, there are almost certainly cases of unauthorized trading that we are unaware of because a trader's doubling-down strategy successfully reversed any losses.)

One thing a bank can do to protect itself is to ensure that the front office (which is responsible for trading) is totally independent of the back office (which is responsible for record keeping and verifying transactions). A more difficult issue is the way in which unauthorized trading is treated when it is uncovered. If a trader conducts unauthorized trading and takes a loss, there are likely to be unpleasant consequences for the trader. But what if the trader makes a profit? It is then tempting to ignore the violations. This is short-sighted, however, because it leads to a culture where risk limits are not taken seriously. This in turn paves the way for disaster.

7.3 BASEL II REGULATIONS

As explained in earlier chapters, the Basel Committee on Banking Supervision develops global regulations, which are then implemented by bank supervisors in each member country. In 1999, it issued an early draft of a regulation that became known as Basel II.6 In part, this was a revision of the methods for calculating credit risk capital (which were discussed in Chapter 6). A surprise inclusion, however, was a clear indication that regulators were planning to require banks to hold capital for operational risk in addition to the capital already required for market risk and credit risk.

Many risk managers deemed capital requirements for operational risk to be unworkable because of the difficulty in quantifying operational risk. However, the Basel Committee recognized that many of the large losses experienced by banks were operational risk losses, not market risk or credit risk losses. Even if operational risks could not be quantified precisely, they considered it important for banks to devote more resources toward managing them.

There has been a parallel development in the regulation of insurance companies. Solvency II, the European Union's regulatory framework for insurance companies issued in 2016, requires capital to be held for operational risk. As mentioned earlier, insurance regulators use a definition of operational risk similar to that used by Basel II.

The result of these regulations has been that operational risk management is given much more emphasis within banks and insurance companies. Operational risk managers are important members of the risk management team at financial institutions. They must understand where losses might occur, which losses should be insured against, and how losses can be mitigated. In addition to requiring capital for operational risk, regulators have also produced guidelines on how operational risk should be managed and have indicated a desire to see evidence that the guidelines are being followed.

It has not been easy for regulators to come up with rules for determining operational risk capital. The final Basel II rules for banks had three approaches:

1. The basic indicator approach,
2. The standardized approach, and
3. The advanced measurement approach (AMA).

Banks typically started by using the basic indicator approach. They then needed to satisfy several criteria to be permitted to use the standardized approach. After that, they could use AMA by satisfying further criteria. The first two approaches are quite simple. The third approach is quite complicated.

In the basic indicator approach, operational risk capital is set equal to 15% of the three-year average annual gross income. Gross income is defined as:

Interest earned - interest paid + non-interest income

The standardized approach is similar, except that separate calculations are carried out by each business line and the percentage applied to gross income varies across business lines. The percentages are in Table 7.1.

Business Line	Capital (% of Gross Income)
Corporate finance	18%
Trading and sales	18%
Retail banking	12%
Commercial banking	15%
Payment and settlement	18%
Agency services	15%
Asset management	12%
Retail brokerage	12%

The AMA in Basel II is much more complicated than the other two approaches. Banks are required to treat operational risk like credit risk and set capital equal to the 99.9 percentile of the loss distribution minus the expected operational loss. The model is illustrated in Figure 7.1.

Under the AMA approach, banks were required to consider every combination of the eight business lines in Table 7.1 and the seven risk types mentioned in the introduction. For each of the 56 (= 7 X 8) combinations, they had to estimate the 99.9 percentile of the ono-year loss. These estimates were then aggregated to determine the total capital requirement.

The Basel Committee has now abandoned AMA and is replacing all three of the approaches in Basel II with a new standardized approach (which will be discussed in the next section).

However, we will later examine some key aspects of the AMA approach because many banks still use it as part of their economic capital determinations. Specifically, the calculation of economic capital requires a probability distribution for the one-year loss and uses the model in Figure 7.1 (but usually with a higher percentile than 99.9%).

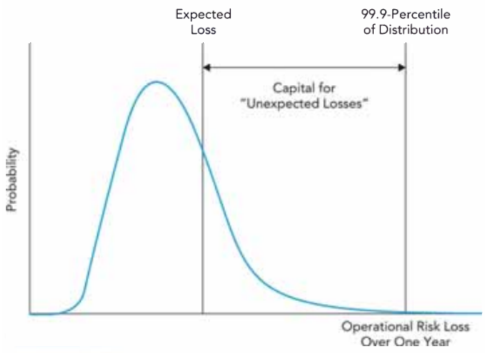


Figure 7.1 The AMA model.

7.4 REVISION TO BASEL II

The AMA operational risk methodology we have described has proved useful in prompting financial institutions to think more about the operational risks they face. However, bank regulators have found the approach unsatisfactory due to the high degree of variation in the calculations carried out by different banks. Two banks presented with the same data were liable to come up with quite different capital requirements under AMA.

The Basel Committee therefore announced in March 2016 its intention to replace all previous approaches for determining operational risk capital with a new approach: the standardized measurement approach (SMA). The SMA first defines a quantity known as the Business Indicator (BI). BI is like gross income, but it is designed to be a more relevant measure of bank size. For example, items such as trading losses and operating expenses, which reduce gross income, are treated differently so that they increase BI.

The BI Component for a bank is calculated from the BI using a piecewise linear relationship. A loss component is then calculated as:

$7X + 7Y + 5Z$

Here, X, Y, and Z are estimates of the average annual loss from operational risk over the previous ten years. The quantity X includes all losses, Y includes only losses greater than EUR 10 million, and Z includes only losses greater than EUR 100 million. The calculations are designed so that the loss component and the BI Component are equal for an average bank. The Basel Committee provides a formula for calculating required capital from the loss component and the BI component.

7.5 DETERMINING THE LOSS DISTRIBUTION

Economic capital calculations require a distribution (like that in Figure 7.1) for several categories of operational risk losses and the combined results. The key determinants of an operational risk loss distribution are

- Average Loss frequency: the average number of times in a year that large losses occur, and
- Loss severity: the probability distribution of the size of each loss.

Loss Frequency

A Poisson distribution is often assumed for loss frequency. This is a distribution of the number of events in a certain time if the events occur a certain rate and are independent of each other. If the expected number of losses in a year is A, the probability of n losses during the year given by the Poisson distribution is

$$\frac{e^{-\lambda}\lambda^{n}}{n!}$$

Table uses the Poisson distribution to give the probability for the number of losses in a year when the average number of losses in the year is 2, 4, and 6.

Loss Severity

The mean and standard deviation of the loss severity is often fitted to a lognormal distribution. This is a distribution where the natural logarithm of the variable is normal. Suppose the mean and standard deviation of the loss size are estimated to be /i and a, respectively. Under the lognormal assumption, the mean of the logarithm of the loss size is

$$ln(\frac{\mu\sigma}{\sqrt{1+w}})$$

and the variance of the logarithm of the loss size is ln (1 + w)

where w = $\left(\frac{\sigma}{\mu}\right)^2$

For example, if the mean and standard deviation of the loss size are estimated (in USD million) as 80 and 40, then w = 0.5^2 = 0.25. The logarithm of the loss size therefore has a mean of:

$ln(\frac{80}{\sqrt{1.25}})$**= 4.27**

and a variance of ln (1.25) = 0.223

Monte Carlo Simulation

Once λ, μ and σ have been estimated, a Monte Carlo Simulation can be used to determine the probability distribution of the loss. The general approach is illustrated in Figure 7.2.

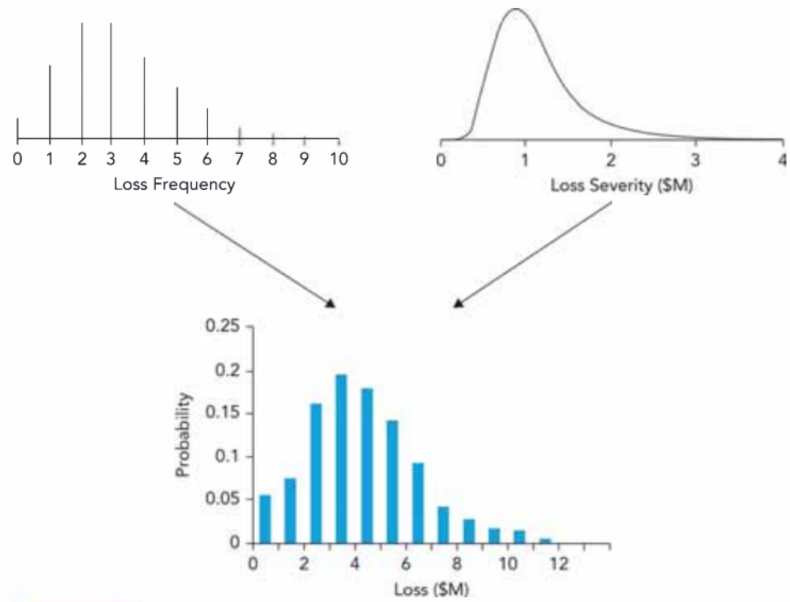


Figure 7.2 **Determination of loss distribution from loss frequency and loss severity.**

The steps in the procedure are as follows.

- Step 1: Sample from the Poisson distribution to determine the number of loss events (= n) in a year.
- Step 2: Sample n times from the lognormal distribution of the loss size for each of the n loss events.
- Step 3: Sum the n loss sizes to determine the total loss.
- Step 4: Repeat steps 1 to 3 many times.

For step 1, we can sample the percentile of the Poisson distribution as a random number between zero and one. For example, suppose that the average loss frequency is 4 and the random number sampled is 0.31.

From Table 7.2, this corresponds to three loss events. This is because the cumulative probability for two loss events or less is

Table 7.2 **Probability Distribution for Number of Losses in a Year for Three Different Values of Average Loss Frequency Parameter, λ**

No. of Losses	Average Loss Frequency		
	2	4	6
0	0.135	0.018	0.002
1	0.271	0.073	0.015
2	0.271	0.147	0.045
3	0.180	0.195	0.089
4	0.090	0.195	0.134
5	0.036	0.156	0.161
6	0.012	0.104	0.161

0.238 (= 0.018 + 0.073 + 0.147) and for three loss events or less is

0.433 (= 0.018 + 0.073 + 0.147 + 0.195).

The sampled number of 0.31 lies between these two cumulative probabilities.

Suppose the loss size has mean 80 and standard deviation 40 (as in the example above). For step 2, we sample three times from a normal distribution with mean 4.27 and variance 0.223.

If the numbers sampled are 4.1, 5.1, and 4.4, the three losses are

$e^{4.1}$= 60.34

$e^{5.1}$= 164.02

$e^{4.4}$ = 81.45

Step 3 gives the total loss on that simulation trial of

305.81 (= 60.34 + 164.02 + 81.45)

By carrying out many Monte Carlo simulation trials such as this, we obtain a probability distribution for the total loss from which the required percentile can be calculated.

Estimation Procedures

Estimating the loss frequency and loss severity for a category of losses involves a combination of data and subjective judgement. Loss frequency should be estimated either from a bank's own data or subjectively by operational risk professionals after careful consideration of the controls in place. We will examine subjective estimation further in the scenario analysis section later on.

When loss severity cannot be estimated from a financial institution's own data, the losses experienced by other financial institutions can sometimes be used as a guide. Mechanisms for sharing loss data between banks have been developed. Additionally, data vendor services (such as Factiva and Lexis-Nexis) can be useful in providing data on publicly reported losses experienced by other banks.

Potential Biases

Data from data vendors can potentially be biased because only large losses are usually reported. If the data from a vendor is used in a direct way to determine the loss severity distribution, the distribution is likely to be biased toward large losses. This bias can be avoided if the data is used to determine only relative loss severity. If data from a vendor indicates Loss Type A (on which a bank has no data) is on average twice as severe as Loss Type B (on which the bank does have data), the bank might assume that the mean loss for Loss Type A is twice that calculated using its own data for Loss Type B. Similarly, if vendor data indicates the standard deviation for Loss Type A is 50% greater than that for Loss Type B, the bank might assume the standard deviation of losses for Loss Type A is 50% greater than the standard deviation of losses for Loss Type B calculated using its own data.

Another potential bias concerns the size of a loss. Suppose Bank B has revenues of USD 20 billion and experiences a loss of USD 300 million. Bank A, with revenues of USD 10 billion, is using this loss event to estimate the severity of a similar loss it might incur.14 Bank A's loss would most likely not be as large as USD 300 million because it is a smaller bank than Bank B. But it would be too optimistic to estimate its loss would be half of that of Bank B (USD 150 million). Shih et al. (2000) use vendor data to estimate a model of the form:

$$\boldsymbol{Estimated\ Loss\ for\ Bank\ A = Observed\ Loss\ for\ Bank\ B} \times \left(\frac{\boldsymbol{Bank\ A\ Revenue}}{\boldsymbol{BankB\ Revenue}}\right)^{\boldsymbol{\beta}}$$

They find that β = 0.23 gives a good fit. The loss for Bank A in our example would therefore be

$$300 \times \left(\frac{10}{20}\right)^{0.23} = 256$$

or USD 256 million. It is also important to adjust loss severity estimates for inflation. A loss of a certain size observed ten years ago can be expected to be larger if the same set of circumstances repeat themselves.

7.5.a Scenario Analysis

Financial institutions also use scenario analysis to estimate loss frequencies and loss severities. It is particularly useful for loss events with a low frequency but high severity. These are the important loss events because they tend to determine the extreme tails of the loss distribution.

The objective for this approach is to list these events and generate a scenario for each one. Some of the scenarios might come from a financial institution's own experience, some might be based on the known experience of other banks, and some might be hypothetical scenarios generated by risk management professionals. Sometimes consultants are used to assist in generating scenarios.

For each scenario, loss frequency and loss severity estimates are made. Monte Carlo simulations (like the one illustrated in Figure 7.2) are used to determine a probability distribution for total loss across different categories of losses. The estimates are usually made by a committee of operational risk experts. The loss frequency estimate should reflect the controls in place at the financial institution and the type of business it is doing.

Estimating the probability of events that happen infrequently is difficult. One approach is to specify several categories and ask operational risk experts to assign each loss to a category. The categories could be

- Scenario happens once every 1,000 years on average ($\lambda = 0.001$),
- Scenario happens once every 100 years on average ($\lambda = 0.01$)
- Scenario happens once every 50 years on average ($\lambda = 0.02$),
- Scenario happens once every ten years on average ($\lambda = 0.1$),
- Scenario happens once every five years on average ($\lambda = 0.2$).

Operational risk experts must also estimate loss severity. Rather than estimate the mean and standard deviation, it might be more appropriate to ask for estimates of the 1 percentile to 99 percentile range of the loss distribution. These estimates can be made to fit to a lognormal distribution. For example, suppose that 20 and 200 are the 1 percentile and 99 percentiles of the loss (respectively). Then ln(20) = 2.996 and ln(200) = 5.298 are the 1 and 99 percentiles for the logarithm of the loss distribution (respectively). From this, it follows that the logarithm of the loss distribution has a mean of (2.996 + 5.298)/2 = 4.147 and a standard deviation of (5.298 - 4.147)/2.326 = 0.49.

The key point here is that scenario analysis considers losses that have never been experienced by a financial institution yet could happen in the future. Managerial judgement is used to assess loss frequency and loss severity. Hopefully, this leads to an active discussion about how such loss events can occur. Scenario analysis can help firms form strategies for responding to a loss event and/or reducing the probability of it happening in the first place.

7.5.b Allocation of Economic Capital

Economic capital is allocated to business units so that a return on capital can be calculated. The procedure for allocating credit risk capital was discussed in Section 6.6. A similar procedure can (in principle) be used for allocating operational risk capital. The allocation of operational risk capital provides an incentive for a business unit manager to reduce operational risk. If the business unit manager can show that he or she has successfully reduced either loss frequency or loss severity, less capital will be allocated to the business unit. The unit's return on capital will then improve and the manager can hope for a bigger bonus. At the very least, the allocation process should sensitize the manager to the importance of operational risk.

Note that it is not always optimal to reduce operational risk. Some level of operational risk is inevitable in any business unit, and any decision to reduce operational risk by increasing operating costs should be justified with a cost-benefit analysis.

7.5.c Power Law

As we saw in Chapter 6, economic capital is often calculated with very high confidence levels. For some probability distributions occurring in nature, it has been observed that a result known as the power law holds. If v is the value of a random variable and x is a high value of v, then the power law holds it is approximately true that:

$$\Pr\,(v > x) \sim K x^{-\alpha}$$

where Pr denotes probability, and K and a are parameters.

The power law describes how fat the right tail of the probability distribution of v is. The parameters K and a depend on the variable being considered. Kis a scale parameter, while a reflects the fatness of the distribution's right tail. As the parameter a decreases, this tail becomes fatter. The power law only describes the right tail of the distribution (not the whole distribution). That is why Equation (7.1) is approximately true only for high values of x (values of x that are well into the right tail of the distribution of v). The power law is based on the work of Polish mathematician D.V. Gnedenko. He showed that the tails of a wide range of distributions share the common properties indicated in Equation (7.1). To be mathematically correct, we should say Equation (7.1) is true for a wide range of distributions in the limit as x tends to infinity. In practice, it is usually assumed to be true for values of x that are in the top 5% of the distribution.

Gnedenko's result has been shown to be true for a wide range of distributions, such as

- The incomes of individuals,
- The magnitude of earthquakes,
- The sizes of cities as measured by population,
- The sizes of corporations,
- The trading volume of a stock,
- The occurrence of a word in text, and
- The number of hits to a website.

As a rough statement, we can say that the power law holds for probability distributions of random variables which are the result of aggregating many independent random effects in some manner.

Work by de Fontnouvelle et al. (2003) suggests the power law holds for operational risk losses. This can be useful in some circumstances. For example, supposed Kand a are estimated as 10,000 and 3 (respectively). Suppose further that we are interested in estimating the 99.5% percentile of the loss distribution (as measured in USD millions). From Equation (7.1), this is the value of x that solves:

$$0.005 = 10000x^{-3}$$

or[21]

$$\ln(0.005) = \ln(10000) - 3\ln(x)$$

so that ln(x) = 4.836 and x = $e^{4.836}$ (or about 126).

7.6 REDUCING OPERATIONAL RISK

Beyond measuring operational risk and determining appropriate capital levels, operational risk units should also try to be proactive in reducing both the probability of large losses and the severity of those losses when they occur. Financial institutions can learn from each other in this area. When a large loss occurs at one financial institution, risk managers throughout the world study what happened and consider the steps that can be taken to avoid a similar loss at their own organization.

7.6.a Causes of Losses

Sometimes operational risk losses can be related to other factors that can be managed. For example, in some situations it might be possible to show that losses can be reduced by increasing employee training or the educational qualifications necessary for a certain position. In other situations, it might be possible to show losses arising from an outdated computer system.

It is not always the case that operational risk losses should be minimized. A cost-benefit analysis should be undertaken because the costs of reducing operational risk can sometimes outweigh the benefits. For example, a study might show that transaction processing errors can be reduced by 5% if a new computer system is developed in conjunction with extra training being given to employees. However, the cost of making the change might greatly exceed the present value of the reduction in losses.

7.6.b Risk Control and Self-Assessment

Risk control and self-assessment (RCSA) is one way in which financial institutions try to understand operational risks while creating an awareness of operational risk among employees. The key term here is self-assessment. Line managers and their staff, not operational risk professionals, are asked to identify risk exposures. The risks considered should include not just losses that have occurred in the past, but those that could occur in the future. There are many RCSA approaches:

- Interviewing line managers and their staff;
- Asking line managers to complete risk questionnaires;
- Reviewing risk incident history with line managers;
- Reviewing third-party reports such as those of auditors, regulators, and consultants;
- Reviewing reports of the experiences of similar managers in other companies;
- Using of suggestion boxes and intranet reporting portals;
- Implementation of a 'whistle blowing' process to encourage the reporting of risk issues; and
- Carrying out brainstorming in a workshop environment.

The assessment process should be repeated periodically (e.g., every year). The frequency of loss events and their severity should be quantified. Some loss events are an inevitable part of doing business. For others, the RCSA process may lead to improvements reducing the frequency of losses, the severity of losses, or both.

7.6.c Key Risk Indicators

A developed understanding of the risks faced by line managers can lead to the development of key risk indicators (KRIs). These are data points that may indicate a heightened chance of operational risk losses in certain areas. In some cases, remedial action can be taken before it is too late. Simple examples of KRIs are metrics related to:

- Staff turnover,
- Failed transactions,
- Positions filled by temps, and
- Unfilled positions.

To use these indicators effectively, it is important to track how they change through time so that unusual behaviour can be identified. Some KRIs are subtler than others. For example, the unwillingness of an employee to take vacations might be an indication that he or she could be engaged in unauthorized trading or embezzling funds. Some organizations have become quite sophisticated, using tools such as surveillance software to search for unusual email or phone activity indicative of an employee engaging in unlawful or unethical activity.

7.6.d Education

Employee education can be important in reducing operational risk. We mentioned earlier how compliance is an area that can lead to huge operational risk losses. Educating employees about unacceptable business practices and (more importantly) creating a risk culture where such practices are perceived to be unacceptable is important. In 2007, Goldman Sachs received adverse publicity when it created a product (called ABACUS) that arguably benefited one client at the expense of another. The firm then took steps to change its risk culture and sent CEO, Lloyd Blankfein, on a 23-country tour of Goldman's regional offices. He spoke to employees about the importance of ethics and emphasized that the company should not sell products to clients unless they fully understand the range of possible outcomes.

Legal disputes are unfortunately an inevitable part of doing business. (This is particularly true in the litigious environment of the United States.) The in-house legal department within a financial institution needs to remind employees to be careful about what they write in e-mails and (when they are recorded) what they say in phone calls.In a legal dispute where an organization is being sued, the organization usually must provide all relevant internal communications. Some can be very embarrassing. For example, Fabrice Tourre, who worked on the ABACUS product at Goldman Sachs, sent the following e-mail to a friend: "More and more leverage in the system. The whole building is about to collapse anytime now . . . Only potential survivor the fabulous Fab . . . standing in the middle of all these complex highly leveraged exotic trades he created without necessarily understanding all the implications of those monstrosities!!!"

Before communicating via the use of emails or recorded phone calls, an employee should always consider whether he or she would be comfortable if the communication became public knowledge.

7.7 INSURANCE

Many operational risks can be insured against. However, operational risk managers need to carefully evaluate whether the cost of insurance can be justified. As explained earlier, the new SMA for assessing operational risk is based on the frequency and magnitude of the losses incurred over the previous ten years. Thus, insuring against a loss can not only reduce the severity of losses, but also reduce capital requirements.

To understand how insurance companies view operational risk, we review the two key risks they face: moral hazard and adverse selection.

7.7.a Moral Hazard

Moral hazard is the risk that the existence of an insurance contract will cause the insured entity to behave in a way that makes a loss more likely. One example of moral hazard concerns rogue trader losses. If an insurance company insures a bank against such losses, it might be concerned traders would take large unauthorized risks. If a gain resulted, the bank would be pleased. If a loss resulted, a claim would be made against the insurance company.

In the light of this type of moral hazard, it is perhaps surprising that it is actually possible to buy insurance against rogue trader losses. In practice, insurance companies manage the moral hazard by carefully specifying how trading limits are implemented and monitored within banks. Rogue trader insurance policies are negotiated by the risk managers and insurance companies often require that the policies are not revealed to traders. Any losses incurred are investigated carefully, and if financial institutions fail to follow their insurance requirements, they might forfeit their payout.

More generally, insurance companies manage moral hazard by using deductibles so that a financial institution is responsible for the first part of any loss. There may also be a co-insurance provision where the insurance company pays only a percentage of a loss rather than the whole amount. Furthermore, there is always a limit on the total amount that can be paid out. Insurance premiums may also increase after a loss has been incurred.

7.7.b Adverse Selection

Adverse selection is the problem an insurance company faces in distinguishing low-risk situations from high-risk situations. If it charges the same premium for a certain type of risk to all financial institutions, it will inevitably attract clients with the highest risk. Consider again the example of rogue trader insurance. If all banks were offered the same insurance premiums, banks with poor internal controls would tend to buy more insurance, while those with good internal controls would consider the cost of the insurance too high (and therefore buy less insurance).

Insurance against cyber risks provides another example of potential adverse selection. Those financial institutions with good cyber defenses are likely to consider cyber insurance to be too expensive, while those that have not invested heavily in this area will find the insurance attractive.

Insurance companies deal with adverse selection by finding out researching potential customers before providing a quote. Car insurance is a good example of this approach. A driver's initial price quote reflects past accidents, speeding tickets, etc. As time goes by, this information is updated, and the insurance premium is adjusted accordingly. In the case of rogue trader insurance and cyber insurance, a financial institution must convince an insurance company that it has good risk controls in place before it can qualify for insurance.

Reading 08 Stress Testing

Learning Objectives

- Describe the rationale for the use of stress testing as a risk management tool.
- Identify key aspects of stress testing governance, including choice of scenarios, regulatory specifications, model building, stress-testing coverage, capital and liquidity stress testing, and reverse stress testing.
- Describe the relationship between stress testing and other risk measures, particularly in enterprise-wide stress testing.
- Explain the importance of stressed inputs and their importance in stressed VaR and stressed ES.
- Identify the advantages and disadvantages of stressed risk metrics.
- Describe the key elements of effective governance over stress testing.
- Describe the responsibilities of the board of directors and senior management in stress testing activities.
- Identify elements of clear and comprehensive policies, procedures, and documentations for stress testing.
- Identify areas of validation and independent review for stress tests that require attention from a governance perspective.
- Describe the important role of the internal audit in stress testing governance and control.
- Describe the Basel stress testing principles for banks regarding the implementation of stress testing.

8.1 INTRODUCTION

Stress testing is a risk management activity that has become increasingly important since the 2007-2008 financial crisis. It involves evaluating the implications of extreme scenarios that are unlikely and yet plausible. A key question for a financial institution is whether it has enough capital and liquid assets to survive various scenarios. Some stress tests are carried out because they are required by regulators. Others are carried out by financial institutions as part of their internal risk management activities.

Earlier chapters have discussed how measures such as value-at-risk (VaR) and expected shortfall (ES) are calculated and used. Stress tests provide additional information for risk managers. When used in conjunction with VaR/ES analyses, they provide a more detailed picture of the risks facing a financial institution. Their advantage is that they can consider the impact of scenarios that are quite different from (and more severe than) the scenarios considered by VaR or ES. As explained in Chapter 2, bank regulators have moved toward basing market risk capital on stressed VaR and (more recently) stressed ES. These are measures based on how market variables behaved during a 12-month period that would have been significantly stressful for a firm's current portfolio.

This chapter discusses how stress-testing scenarios are generated internally by financial institutions. It also covers regulatory requirements, stress-testing principles published by the Basel Committee, and governance issues.

8.2 STRESS TESTING VERSUS VAR AND ES

VaR and ES are based on the estimated loss distribution. VaR allows a financial institution to reach a conclusion in the form of we are X percent certain that our losses will not exceed the VaR level during time T. In the case of ES, the conclusion is if our losses do exceed the VaR level during time T, the expected (i.e., average) loss will be the ES amount.

One disadvantage of VaR and ES is that they are usually backward-looking. They assume the future will (in some sense) be like the past. Stress testing, however, is designed to be forward-looking and answer more general "What If?" questions. Unlike VaR and ES, stress testing does not provide a probability distribution for losses. While it may be possible for management to assess probabilities for different scenarios, using stress tests to derive the full range of all possible outcomes is not usually possible.

Risk managers therefore have two types of analyses available to them. One is a backward-looking analysis where a loss distribution can be estimated. The other is a forward-looking analysis where different scenarios are assessed. The backward-looking VaR/ES analysis looks at a wide range of scenarios (some good for the organization and some bad) that reflect history. On the other hand, stress testing looks at a relatively small number of scenarios (all bad for the organization). There are other differences between the stress-testing approach and VaR/ES analyses. In the case of market risk, the VaR/ES approach often has a short time horizon (perhaps only one day), whereas stress testing usually looks at a much longer period.

The objective in stress testing should be to obtain an enterprise-wide view of the risks facing a financial institution. Often, the scenarios are defined in terms of macroeconomic variables such as GDP growth rates and unemployment rates. The impact of these variables on all parts of the organization must be considered along with the interactions between different areas. The overarching objective in stress testing is to determine whether the financial institution has enough capital and liquidity to survive adverse events.

Stressed VaR and Stressed ES

The distinction between stress testing and VaR/ES measures is blurred by measures known as stressed VaR and stressed ES. These have been mentioned in earlier chapters and will now be reviewed again.

VaR and ES have traditionally been calculated using data over from the preceding one to five years. Daily movements in risk factors during this period are used to calculate potential future movements. In stressed VaR and stressed ES, however, this data is gathered from particularly stressful periods.2 Stressed VaR and stressed ES therefore produce conditional loss distributions and conditional risk measures. Specifically, they are conditional on a repeat of a given stressed period and can be considered a form of historical stress testing.

Although stressed VaR/ES and stress testing have similar objectives, there are important differences between them. Suppose the year 2008 is used as the stressed period. Stressed VaR would reach the conclusion:

If we had a repeat of 2008, we are X% certain that losses over a period of T days will not exceed the stressed VaR level.

Stressed ES would reach the conclusion:

If losses over T days do excess the stressed VaR level, the expected (i.e., average) loss is the stressed ES.

Typically, Tis a short period (i.e., one to ten days). In contrast, stress testing usually has a longer time horizon. It seeks to answer the questions such as, "If the next year is a repeat of 2008, how would our organization survive?" or "If next year were like 2008 but twice as bad, how would we survive?" It does not consider what would happen during the worst T days of 2008. Rather, it considers the impact of the whole of 2008 being repeated.

Traditional VaR measures are designed to quantify the full range of possible outcomes and can therefore be backtested. For example, suppose we have a procedure for calculating one-day VaR with 99-percent confidence. We can test this measuring by seeing how well the procedure would have worked in the past. If we find that losses would have only exceeded the calculated VaR around 1% of the time, we can have some confidence in our results. However, it is not possible to back-test stressed VaR or the output from stress testing in this way because these measures focus on extreme outcomes, which we do not expect to observe with any particular frequency.

8.3 CHOOSING SCENARIOS

The first step in choosing a stress-test scenario is to select a time horizon. While one-day or one-week scenarios are occasionally considered, scenarios lasting three months to two years are more common. The time horizon should be long enough for the full impact of the scenarios to be evaluated, and very long scenarios can be necessary in some situations. For example, a pension plan or insurance company concerned about longevity risk might consider stress tests stretching over several decades.

As we will describe later, some scenarios are determined by regulators. At this stage, we focus on those that are chosen internally. We will explain several ways in which these scenarios can be generated.

8.3.a Historical Scenarios

Scenarios are sometimes based on historical data, and it is assumed that all relevant variables will behave as they did in the past. When discussing the historical simulation approach to determining VaR and ES, we explained that for some variables (e.g., equity prices and exchange rates) it is appropriate to create a scenario that assumes the proportional changes observed in the past are repeated. For other variables (e.g., interest rates and credit spreads), it is appropriate to assume the actual changes from the past are repeated. A similar point applies here. Actual changes from the stressed period will be assumed to recur for some variables, while proportional changes will be assumed for others.

There are many historical scenarios that might be of concern to risk managers. The 2007-2008 U.S. housing-related recession, which led to serious problems for many financial institutions, is an obvious one to use. Scenarios that could give rise to other adverse outcomes include the plunge in oil prices seen in the second half of 2014 and the flight to quality following Russia's default on its bonds in August 1998.

Sometimes, a moderately adverse scenario from the past is made more extreme by multiplying the movements in all risk factors by a certain amount. For example, we could take what happened during a certain loss-making six-month period in the past and double (or triple) the movements in all relevant variables. When magnified in this way, the scenario might become a much more serious problem for a financial institution. However, this approach assumes there is a simple linear relationship between the movements in risk factors. This is not necessarily the case, however, because correlations between risk factors tend to increase as economic conditions become more stressed.

Sometimes, historical scenarios are based on what happened to all market risk factors over one day or one week. For example, the impact of a day like October 19, 1987 (when the S&P 500 fell by 22.3 standard deviations) could be assessed. If this is considered too extreme, a scenario could be created from the days around January 8, 1988 (when the S&P 500 fell by 6.8 standard deviations). Other dates with large movements in equity prices are September 11, 2001 (during the 9/11 terrorist attacks) and September 15, 2008 (when Lehman Brothers declared bankruptcy). For a big one-day movement in interest rates, April 10, 1992 (when ten-year bond yields moved by 8.7 standard deviations) could be used.

These short-horizon stress tests can be supplements to stressed VaR and stressed ES calculations. While stressed VaR and stressed ES consider extreme movements during just one stressed period, short-horizon stress tests can pick big movements from many different stressed periods in the past.

8.3.b Stress Key Variables

One approach to scenario building is to assume that a large change takes place in one or more key variables. Changes that might be considered include:

- A 200-basis point increase in all interest rates,
- A 100% increase in all volatilities,
- A 25% decline in equity prices,
- A 4% increase in the unemployment rate, and
- GDP declining by 2%. Other changes could involve factors such as exchange rates, commodity prices, and default rates.

For market risks, a financial institution's internal systems will provide the impact of relatively small changes in the form of Greek letters such as delta, gamma, and vega (which are discussed in later chapters). In the case of stress testing, however, the changes are so large that these measures cannot be used. Furthermore, whereas Greek letters quantify the risks arising from changes to a single market variable over a short period of time, stress testing often involves the interaction of several market variables over much longer periods.

8.3.c Ad Hoc Stress Tests

The stress tests we have described so far are likely to be carried out on a regular basis (e.g., every month) and the results can provide a financial institution with a good indication of the robustness of its financial structure. But it is important for firms to develop other scenarios reflecting current economic conditions, the particular exposures of the financial institution, and an up-to-date assessment of possible future adverse events. History never repeats itself exactly, and managerial judgement is necessary to either generate new scenarios or modify existing scenarios based on past data.

The decision by the U.K. government to hold a vote on whether to leave the European Union could have led to an ad hoc stress test risk for a financial institution with major business interests in the U.K. There was no historical precedent for the vote and so historical scenarios would not have captured the risks involved. Prior to the vote in June 2016, most people did not expect a "leave" decision despite the fact that it was a plausible outcome. A scenario where the U.K. population votes to leave the European Union would therefore have constituted a valid stress scenario for a financial institution prior to June 2016.

Other ad hoc stress tests could consider the impact of a change in government policy on a key issue affecting a financial institution or a Basel regulation that would require more capital to be raised in a short period of time. Adverse scenarios suggested by professional economists should be considered carefully. In 2005-2006, many economists suggested (correctly as it turned out) that the U.S. housing market was experiencing a bubble that sooner or later would burst. Even if the board and senior management at a financial institution did not agree with this assessment, it would have made sense to use it as a stress scenario.

The boards, senior management, and economics groups within financial institutions are in a good position to use their understanding of markets, world politics, and current global uncertainties to develop adverse scenarios. One way of developing the scenarios is for a committee of senior management to engage in brain-storming sessions. Research suggests committees consisting of three to five members with different backgrounds work best.3 A key role of the committee should be to recommend actions that can be taken to mitigate unacceptable risks.

8.3.d Using the Results

It is important that senior management recognizes the importance of stress testing and incorporates it into its decision making. We will discuss the role of the board and the governance of stress testing later. At this stage, we note that involving senior management in building scenarios makes it more likely for the stress testing to be taken seriously and used for decision-making.

It should be emphasized that the purpose of stress testing is not just to produce output answering, "What if?" questions. Senior management and the board should carefully evaluate stress-test findings and decide whether some form of risk mitigation is necessary. There is a natural human tendency for a decision-maker to base decisions on what he or she considers to be the most likely outcome and to regard alternatives to be so unlikely that they are not worth considering.4 Stress testing should be used by the board and senior management to ensure that this does not happen.

8.4 MODEL BUILDING

It should be possible to observe how most of the relevant risk factors behaved during the stressed period when building a scenario. The impact of the scenario on a firm's performance can then be assessed in a fairly direct way. However, it may be necessary to use judgement in determining the ease with which the firm could raise more capital or improve its liquidity.

Scenarios constructed by stressing key variables (and ad hoc scenarios) typically specify movements in only a few key risk factors or economic variables. To complete the scenarios, it is necessary to construct a model to determine how a range of other variables can be expected to behave. The variables specified in the scenario definition are sometimes referred to as core variables, whereas the other variables are referred to as peripheral variables.

One approach is to carry out an analysis (e.g., linear regression) relating the peripheral variables to the core variables. However, it is important to recognize that the focus is on the relationship between variables in stressed market conditions (rather than normal market conditions). Stressed periods from the past are therefore likely to be most useful in determining the relevant relationships.

For credit risk losses, data provided by rating agencies can be useful. Table 6.1, for example, shows data on the annual percentage default rates for all rated companies between 1970 and 2016. This can be related to economic variables (e.g., the GDP growth rate and unemployment rate) to determine the overall default rates that can be expected in different scenarios. This can then be scaled up or down to estimate default rates for the different categories of loans on a financial institution's books. A similar analysis can be carried out for recovery rates so that loss rates can be determined.

For assessing market risk losses, the relevant peripheral variables are likely to be those whose movements can be related to changes in core risk factors (e.g., interest rates and equity prices). In areas such as investment banking, profitability is likely to be related to equity prices and key economic variables such as GDP growth.

Knock-On Effects

Analysts should consider not only a scenario's immediate consequences, but also what are referred to as knock-on effects. A knock-on effect reflects the impact of how firms (particularly other financial institutions) respond to an adverse scenario. In responding to the adverse scenario, the companies often take actions exacerbating adverse conditions.

Consider a scenario that might have been constructed around a possible US housing price bubble in 2005-2006. It could have been assumed house prices would decline by 5-10%, which in turn would have increased the loss on a bank's mortgage portfolio. In fact, the scenario led to much more severe outcomes as outlined below:

- Some houses were worth less than their outstanding mortgage. Even though the owners could afford to service a mortgage, many chose to default. In effect, they exercised an option to sell the house back to the lender for the amount outstanding on the mortgage.7 The house was then sold by the lender. This increased the supply of houses on the market, making the decline in housing prices greater than it otherwise would have been. This increased the losses on mortgages and securities created from mortgages.

- There was a flight to quality where all risky assets were perceived to be less attractive. As a result, equity prices and corporate bond prices declined sharply. The decline in corporate bond prices meant credit spreads increased.
- Banks were concerned about the creditworthiness of other banks and were reluctant to engage in interbank lending. This increased funding costs for banks.

8.5 REVERSE STRESS TESTING

Stress testing involves constructing scenarios and then evaluating their consequences. Reverse stress testing takes the opposite approach: It asks the question, "What combination of circumstances could lead to the failure of the financial institution?"

One reverse stress-testing approach involves the use of historical scenarios. Under this approach, a financial institution would look at a series of adverse scenarios from the past and determine how much worse each scenario would have to be for the financial institution to fail. For example, it might conclude that a recession three times worse than the one in 2007-2008 would lead to failure. As already mentioned, simply multiplying the changes observed in all risk factors during the 2007-2008 recession by the same amount is an approximation because there are likely to be non-linearities. Ideally, a firm would use a more sophisticated model incorporating the tendency for correlations to increase as market conditions become more stressed.

Analyzing all risk factors to find a plausible combination leading to firm failure is not usually feasible. One approach is to define a handful of key factors (e.g., GDP growth rate, unemployment rate, equity price movements, and interest rates changes) and construct a model relating all other relevant variables to them. It is then possible to search iteratively over all factor combinations to determine scenarios leading to failure.

Reverse stress testing can be an input to the work of a stress testing committee. The committee is likely to discard some of the scenarios generated by reverse stress testing as totally implausible while flagging others for further investigation.

8.6 REGULATORY STRESS TESTING

Up to now, our discussion has centred around stress tests designed by financial institutions themselves. Regulators in many jurisdictions (including the United States, the United Kingdom, and the European Union) also require banks and insurance companies to carry out specified stress tests. In the United States, for example, the Federal Reserve carries out a stress test of all banks with consolidated assets of over USD 50 billion. This is referred to as the Comprehensive Capital Analysis and Review (CCAR). Banks are required to consider four scenarios:

1. Baseline,
2. Adverse,
3. Severely adverse, and
4. An internal scenario.

In early 2018, the Federal Reserve described the scenarios planned for 2018:

For the 2018 cycle, the severely adverse scenario is characterized by a severe global recession in which the U.S. unemployment rate rises almost 6 percentage points to 10 percent, accompanied by a steepening Treasury yield curve. The adverse scenario features a moderate recession in the United States, as well as weakening economic activity across all countries included in the scenario.

The adverse and severely adverse scenarios describe hypothetical sets of events designed to assess the strength of banking organizations and their resilience. They are not forecasts. The baseline scenario is in line with average projections from surveys of economic forecasters. It does not represent the forecast of the Federal Reserve.

Each scenario includes 28 variables-such as gross domestic product, unemployment rate, stock market prices, and interest rates--encompassing domestic and international economic activity. Along with the variables, the Board is publishing a narrative that describes the general economic conditions in the scenarios and changes in the scenarios from the previous year.

Banks must submit a capital plan, documentation to justify the models they use, and the results of their stress tests. If they fail the stress test because their capital is insufficient, they are likely to be required to raise more capital and restrict the dividends they can pay until they have done so.

Banks with consolidated assets between USD 10 billion and USD 50 billion are subject to the Dodd-Frank Act Stress Test (DFAST). The scenarios in DFAST are like those in CCAR. However, banks are not required to submit a capital plan (as capital management is based on a standard set of assumptions).

By choosing the scenarios, bank regulators can evaluate the ability of different banks to survive adverse conditions in a consistent way. But they make it clear that they also want to see scenarios developed by the banks themselves that reflect their particular vulnerabilities.

8.7 GOVERNANCE

Governance is an important part of stress testing. The governance process should determine the extent of the stress testing carried out by a financial institution. It should also ensure that the assumptions underlying the tested scenarios have been carefully thought out, that the results are prudently considered by senior management, and that actions based on the results are taken when appropriate. The Board and Senior Management The governance structure within a financial institution is likely to depend on the legal, regulatory, and cultural norms within a country. Generally, there should be a separation of duties between the board of directors and senior management. The board of directors has the responsibility to oversee the key strategies. It is also responsible for the firm's risk appetite (i.e., the amount and type of risk an organization is willing to take to meet its strategic objectives) and risk culture (i.e., the financial institution's norms along with the collective attitudes and behaviours of its employees).

Stress testing is an important way in which risks are assessed within an organization. The board should define how stress testing is carried out. Specifically, it should determine the procedures used to create the scenarios as well as the way in which assumptions and models are used to evaluate them.

Board members do not carry out stress testing themselves, but they should be sufficiently knowledgeable to ask penetrating questions. They should feel free to use their own experience and judgement to ask for changes in the assumptions underlying the scenarios (or even to ask for totally new scenarios to be considered). When key decisions to mitigate risks are required, the board should feel free to ask for other analyses to supplement the stress-testing results.

Senior management is responsible for ensuring the stress testing activities authorized by the board are carried out by competent employees as well as periodically reporting on those activities to the board. Senior management is also responsible for ensuring the organization is adhering to the appropriate policies and procedures.

It is tempting to use the same scenarios each time a stress test is carried out. However, senior management should ensure that the scenarios change as the economic environment changes and as new risks appear on the horizon. Stress testing should not be done mechanically just to satisfy the board and regulators. It should be an important part of the firm's decision-making and risk-mitigation strategies.

Senior management should have a deep understanding of how stress tests are carried out and should be in an even better position than the board to challenge key assumptions and models (or suggest new scenarios for consideration). Stress testing should not be done in a routine way using the same set of assumptions each time. Rather, the methods used should be refined over time. Even if the nature of a scenario is not changed, consideration should be given to changing its severity in light of changing circumstances. For example, a scenario where there is a 20% decline in equity prices might be changed to one where there is a 30% decline as volatilities increase.

It is important for the board and senior management to ensure stress testing covers all business lines and exposures. The same scenarios should be used across the whole financial institution, and the results should then be aggregated to provide an enterprise-wide view of the risks. Sometimes there will be offsets, but a scenario that leads to losses in one part of the business can do so in other parts as well. A range of different time horizons should be also considered because some adverse scenarios materialize more quickly than others.

Financial institutions must keep sufficient capital and liquid assets to survive stressful situations. Key outputs from a stress test are therefore the scenario's impact on capital and liquidity. Senior management and the board should carefully consider whether the results of stress tests indicate that more capital should be held or that liquidity should be improved. They should keep in mind that once an adverse scenario is underway, they are likely to have much less flexibility in managing capital and liquidity.

8.7.a Policies and Procedures

A financial institution should have written policies and procedures for stress testing and ensure that they are adhered to. These policies and procedures should be clearly stated and comprehensive to ensure that different parts of the organization approach stress testing in the same way. The policies and procedures should

- Describe why stress testing is carried out,
- Explain stress-testing procedures to be followed throughout the company,
- Define the roles and responsibilities for those involved in stress testing,
- Define the frequency at which stress testing is to be performed,
- Explain the procedures to be used in building and selecting scenarios,
- Explain how independent reviews of the stress-testing function will be carried out,

- Provide clear documentation on stress testing to third parties (such as regulators, external auditors, and rating agencies) as appropriate,
- Indicate how the results of stress testing are to be used and by whom,
- Be updated as appropriate as stress-testing practices will change as market conditions change,
- Allow management to track how the results of stress tests change through time, and
- Document the operation of models and other software acquired from vendors or other third parties.

Documenting activities within a financial institution is often not a popular task. It is usually viewed as a less interesting and less creative activity than, say, building a model to investigate the impact of a recession where GDP declines. However, documentation is important so far as it ensures continuity if key employees leave and satisfies the needs of senior management, regulators, and other external parties.

8.7.b Validation and Independent Review

Stress-testing governance should include independent review procedures. The reviews themselves should be unbiased and provide assurance to the board that stress testing is being carried out in accordance with the firm's policies and procedures. In this context, it is worth noting that a financial institution uses many different models that are required (whether they are part of the stress-testing procedures or not) to be subject to independent review to ensure that they are operating as intended.

It is important that the reviewers of stress-testing procedures be independent of the employees conducting the stress test. The review should

- Cover the qualitative or judgemental aspects of a stress test,
- Ensure that tests are based on sound theory,
- Ensure that limitations and uncertainties are acknowledged, and
- Monitor results on an ongoing basis.

It is also important to ensure models acquired from vendors are subject to the same rigorous review as internal models.

The validation of stress-testing models is more difficult than the validation of other models because stress testing deals with rare events. As mentioned earlier, a VaR model with a one-day time horizon and 99-percent confidence level can be validated by counting the percentage of times actual losses would have exceeded the VaR level if the model had been used in the past. There is no similar way of validating the output from a stress test. Other validation approaches are also difficult because the limited amount of data available from previous stressed situations.

As we have explained, models describing the relationship between variables in normal market conditions may not describe how they behave in stressed market conditions. For example, correlations tend to increase in stressed market conditions and recovery rates tend to decline. The independent review should ensure that these phenomena are incorporated into stress testing models.

The independent review should reach conclusions on the conceptual soundness of the stress-testing approach. When there are doubts about the best model to use in a certain situation, the results from several models can be compared and the totality of stress-testing reports can make the attendant uncertainties clear. It is often more appropriate to provide a range of possible losses rather than a single estimate. Expert judgement should be used to ensure results are presented in a way that is most useful for decision-making.

8.7.c Internal Audit

The internal audit function has an important role to play in stress-testing governance. It should ensure that stress tests are carried out by employees with appropriate qualifications, that documentation is satisfactory, and that the models and procedures are independently validated. The internal audit function is not responsible for conducting the stress testing itself. Instead, it assesses the practices used across the whole financial institution to ensure they are consistent. Sometimes it will be able to find ways in which governance, controls, and responsibilities can be improved. It can then provide advice to senior management and the board on changes it considers to be desirable.

8.8 BASEL STRESS-TESTING PRINCIPLES

The publications of the Basel Committee have emphasized the importance of stress testing. The Basel Committee requires market risk calculations based on internal VaR and ES models to be accompanied by "rigorous and comprehensive" stress testing. Similarly, banks using the internal ratings-based approach in Basel II to determine credit risk capital are required to conduct stress tests to assess the robustness of their assumptions.

In May 2009, the Basel Committee published stress-testing principles for banks and their supervisors.10 The principles were very much influenced by the 2007-2008 crisis, and they emphasize the importance of stress testing in determining how much capital is necessary to absorb losses from large shocks.

The principles note that stress testing plays an important role in:

- Providing forward-looking assessments of risk,
- Overcoming the limitations of models and historical data,
- Supporting internal and external communications,
- Feeding into capital and liquidity planning procedures,
- Informing and setting of risk tolerance, and
- Facilitating the development of risk mitigation or contingency plans across a range of stressed conditions.

Banks were arguably lulled into a false sense of confidence by the quiet economic conditions in the years preceding the crisis. It is therefore not surprising that the Basel Committee considers stress testing particularly important after long periods of benign conditions. The crisis showed that such conditions can lead to complacency and the under-pricing of risk.

In examining the shortcomings of the stress testing carried out prior to the 2007-2008 crisis, the Basel Committee reached several conclusions. They can be summarized as follows.

- The involvement of the board and senior management is important. Top management and board members should be involved in setting stress-testing objectives, defining scenarios, discussing the results of stress tests, assessing potential actions, and decision-making. The Basel Committee notes that the banks that fared well in the financial crisis had a senior management that took an active interest in the development and operation of stress testing, with the results of stress testing serving as an input into strategic decision-making. At some banks, stress testing was a mechanical exercise that did not influence decision-making to any great extent and did not take account of changing business conditions. Sometimes, stress tests were carried out within business lines without interactions being considered and enterprise-wide results being produced. Banks should have the ability to aggregate exposures and respond quickly as problems emerge.

- The stress-testing methodologies used at some banks did not enable exposures in different parts of the bank to be aggregated. Experts from different parts of the banks did not cooperate in producing an enterprise-wide risk view. For example, it would have made sense for the optimism of the mortgage-backed securities traders to be tempered by retail lenders. The methodologies assumed average relationships between risk factors that had been observed in the past could be expected to continue to hold in the future. The knock-on effects that were mentioned earlier in this chapter were not considered.

- The scenarios chosen in the stress tests proved to be too mild and had durations that were too short. Additionally, they underestimated the correlations between different risk types, products, and markets. There was too much reliance on historical scenarios and not enough consideration of the risks created by introduction of new products and the new positions taken by the banks.

- Particular risks were not covered in sufficient detail in the scenarios. For example, risks relating to structured products, products awaiting securitization, imperfect hedging, and counterparty credit risk were not fully considered. The impact of a stressed scenario on liquidity was underestimated. The crisis gave rise to systemic risks; banks hoarded liquidity and were unwilling to advance loans to other banks in the way they would in normal market conditions

After observing how stress testing had evolved since the crisis, the Basel Committee published a consultative document with a revised set of principles in 2017.11 These may be used by national authorities to design stress-testing rules, guidance, or principles. The revised principles are summarized in the appendix and are consistent with the points we have made in this chapter.

Reading 09 Pricing conventions, Discounting and Arbitrage

Learning Objectives

- Define discount factor and use a discount function to compute present and future values.
- Define the "law of one price," explain it using an arbitrage argument, and describe how it can be applied to bond pricing.
- Identify arbitrage opportunities for fixed income securities with certain cash flows.
- Identify the components of a US Treasury coupon bond, and compare the structure to Treasury STRIPS, including the difference between P-STRIPS and C-STRIPS.
- Construct a replicating portfolio using multiple fixed income securities to match the cash flows of a given fixed-income security.
- Differentiate between "clean" and "dirty" bond pricing and explain the implications of accrued interest with respect to bond pricing.
- Describe the common day-count conventions used in bond pricing.

Note: Some of the topics as per learning objectives are already covered in Book 3 Properties of interest rate and hence not covered again in this topic. It is advised to revise Properties of interest rate topic before preparing this topic. Repeated discussions LOs are market red.

9.1 INTRODUCTION

In properties of interest rate reading we learned how can we derive discount rates for various periods from Treasury bonds using bootstrapping method. This chapter in curriculum starts from the explaining the concept of bootstrapping interest rates from Treasury bonds. Because we already covered this concept in aforementioned reading and there is no change in final message delivered, we will skip the discussion of bootstrapping. The discount rates (which is used to create term structure) derived from the Treasury bonds can be applied to price other bonds with similar features. This is possible because of law of one price.

Law of one price

The law of one price says, two securities with same cashflows (in value and timing) should be priced at same rate because investor does not care about the source of cash flows as long as the other factors like liquidity and risks are same for both the securities. Let's say, you have choice of two securities A and B, both having same face value with coupon rate of 5% semi annual and time to maturity is 5 years. Security A is currently trading at $98.56, how much will you be willing to pay for security B. Obvious answer is same price as security A. The same principle is applied by all market participants and hence price of both securities should be aligned.

The better explanation for 'same price of two securities (or portfolios) providing same cash flow' is provided by arbitrage principle. However, mathematics of both 'law of one price' and 'arbitrage principle' is same, hence we prefer using law of one price as a shortcut for securities pricing. Please note that the bond is very complex securities and pricing of these securities depends on lots of other factors. In this reading to simplify our discussion we will ignore other factors which affect price of bonds securities. In real life you might see law of one price is not working, the reason might be other factors which are not discussed in this reading. We will learn more about these factors in FRM Part II.

What if two assets providing the same cashflows at same point in time are not selling at same price?

Consider the two portfolios A and B providing same cash flows and portfolio A which is priced higher than the portfolio B. This is an arbitrage opportunity and traders will start selling portfolio A which has higher price and buy portfolio B. If trader does not own any portfolio then he will short sell A and long B and will be able to lock risk free profits. This will keep happening till both the portfolios are priced equally.

9.2 DISCOUNT RATE CALCULATION

This is similar to bootstrapping method we discussed in Properties on interest rate reading. However, the difference is, we only focus on discount rates rather than interest rate calculation from discount rates. We will use exactly same illustration discussed in Properties of interest rate reading so that it is easy to compare with bootstrapping.

Discount rates are is derived using Treasury bonds which can be used to price other security. Assume a Treasury Bond paying semiannual coupon at 5% and has 1 year time to maturity. We can assume this as package of zero-coupon bond by treating 6 months coupon as ZCB with face value of 2.5 and 12 months coupon + face value as ZCB with face value of 102.5 maturing in 12 months.

In bootstrap method spot rates are derived progressively using coupon paying Treasury bonds.

Illustration:

Time To maturity	Coupon Semi annual	Price	Face Value
Bond A: 6 months	5%	100.5	100
Bond B: 12 months	6%	104.2	100

Solution:

Step 1: First we will use Bond A to derive discount factor for 6 months and then this discount rate will be used in Bond 2 to derive discount factor for 1 year.

Equation A (Bond A): 100.5 = 102.5 X ($d_{0.5}$)

Equation B (Bond B): 104 = 3 X ($d_{0.5}$) + 103 x (d_1)

Where, d0.5 is discount factor of 6 months and d1 is discount factor of 1 year. First we will use equation A to calculate d0.5, then we will use this d0.5 in equation B to calculate d1.

Equation A : d0.5 = 100.5 / 102.5 = 0.9804

Equation B: 104 = 3 X 0.9804 + 103 X (d1)

d1 = 0.9811

9.3 REPLICATING BOND CASH FLOW PORTFOLIO

We discussed in law of one price, if two portfolios provide same cashflows then it should be priced at same level. In this section we will see how to create portfolio which mimicking the cash flow of another portfolio. Assume we have Portfolio Green consists of a bond (Face value $100) currently priced at $105.64 , pays coupon of 8% semi annual and time to maturity is 2 years (4 periods). (See diagram given below). Assume we have 4 bonds (given in table below) to create replicating portfolio call it a yellow portfolio which mimics the cash flow of green portfolio. Because both the portfolios provide same cashflows, we can say the price should be same.

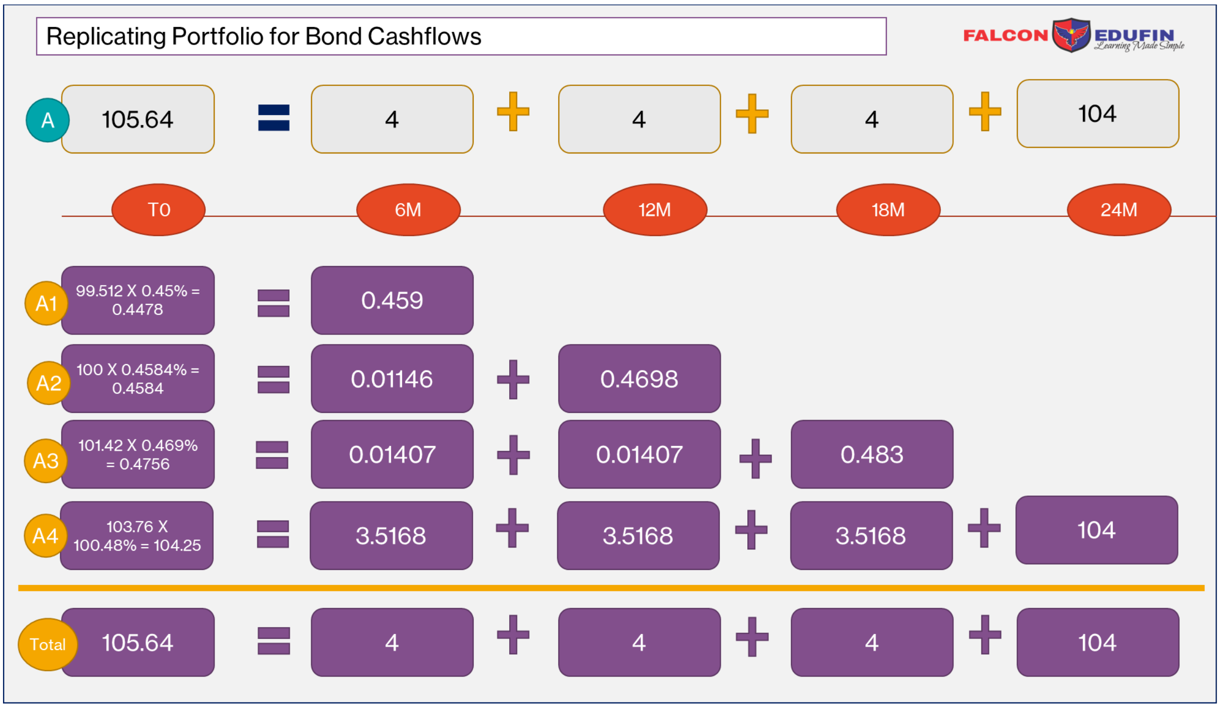
Note: For the pricing of these bonds semi annual yield of 5% and face value of $100 is assumed and all prices can be derived using TI BA II Plus calculator TVM Function.

	Bond	Coupon Semi A	Time To Maturity	Price
Green Portfolio	Bond A	8%	2 years	$105.64
Yellow Portfolio	Bond A1	4%	0.5 year	$99.512
	Bond A2	5%	1 year	$100
	Bond A3	6%	1.5 years	$101.42
	Bond A4	7%	2 years	$103.76

Note: Green portfolio and yellow portfolios are indicated by same color labels (of bond name) in following figure for easy review.

Following diagram shows the percentage of face value required from each bond to create a replicating portfolio(yellow) which mimics the cash flow of green portfolio. In this section we will learn how to calculate the percentage of face value in later part. First lets focus on how the replicating portfolio works. We have bond A which pays coupon of 8% semi annual, which means after every 6 months coupon of $4 will be received and at the end of 24 months coupon with face value amounting to $104 is received. Lets consider the last cash flow of $104 from green portfolio. If we have to mimic this cash flow the only option we have is Bond A4 which matures in 24 months. Bond A1 to A3 are of no use because these bonds are maturing before 24 months. We can see in the following diagram Bond A4 needs 100.48% of face value to get same cash flow as Bond A's at the end of 24 month. Bond A4s cash flow at the end of 24th month with face value of $100 is 103.5. But we made investment equal to 100.48% of face value, hence Bond A4s cash flow on invested amount is $103.5 X 100.48% = $104. This cash flow is same as cash flow from green portfolio of Bond A in 24th month. However, the investment in Bond A4 is generating cash flow for all periods including periods prior to 24th month which creates overlap with other Bonds A1 to A3.

Now consider 18th month cash flow. In this month the combined cash flow $4 from Bond A3 and A4 is equal to cash flow of Bond A. In this case Bond A4 substantially contributes to the cash flow. Total investment in Bond A3 is 0.469% of face value. Hence cash flow from Bond A3 in 18th month is $0.483 (0.469% of $103.00)



Bond A4: To calculate the percentage of face value (FA4) we have to start with Bond A4. Best method is to start with matching cash flow of maturity month of every bond. Hence for bond A4 we will match the cash flow of Bond A = $104 and Bond A4 = FV4 X 103.5.

FA4 X 103.5 = 104 X 100, Solving this equation gives us

FA4 = 100.48%

104 is multiplied by 100 to calculate percentage of face value. Same will be applicable for all the following calculations.

For Bond A3, we have to match only coupon cash flow from bond A = \$4 in 18^{th} month with cash flow of Bond A3 face value cash flow 103. However, we also have cash flow from Bond A4 in 18^{th} month which needs to be adjusted in equation.

FA3 X 103 + 100.48% of 3.5 = \$4 X 100

FA3 = 0.469%

Similarly for Bond A2:

FA2 X 102.5 + 3 X 0.469% + 3.5 X 100.48% = 4 X 100

FA2 = 0.4584%

For Bond A1

FA1 X 102 + 2.5 X 0.4584% + 3 X 0.469% + 3.5X 100.48%

FA1 = 0.45%

To calculate cash flows for respective bonds, simply multiply every bonds cash flow with percentage of face value. Also the investment value is price of each bond multiplied by percentage of investment calculated above.

Arbitrage opportunity

If say green portfolio is selling at lower price than yellow portfolio, arbitrage exists. To exploit the arbitrage opportunity trader will take the long position in cheaper portfolio and short position in expensive portfolio to generate fixed risk free profit. This will create demand side pressure for cheaper portfolio and supply side pressure for expensive portfolio, will result in price increase cheaper portfolio and price decrease for expensive portfolio. Hence, the arbitrage will force the price of both the portfolios to be equal.

9.4 COMPONENTS OF US TREASURY COUPON BONDS

STRIPs are created by dealers when coupon bearing bond is delivered to the Treasury. Bond is exchanged for its principal and coupon components known as C Strip and P Strip respectively. We can simply think of C Strip and P Strip as zero coupon bonds with face value equal to coupon and principal respectively. P strips are also known as TPs or Ps and C strips are also known as TINTs or INTs (Short form of interest). Mispricing of C Strip and P strip results into arbitrage, however, transaction costs discourage the arbitrage.

Reading 10 Interest Rates

AFTER COMPLETING THIS READING YOU SHOULD BE ABLE TO:

- CALCULATE AND INTERPRET THE IMPACT OF DIFFERENT COMPOUNDING FREQUENCIES ON A BOND'S VALUE.
- DEFINE SPOT RATE AND COMPUTE SPOT RATES GIVEN DISCOUNT FACTORS.
- INTERPRET THE FORWARD RATE, AND COMPUTE FORWARD RATES GIVEN SPOT RATES.
- DEFINE PAR RATE AND DESCRIBE THE EQUATION FOR THE PAR RATE OF A BOND.
- INTERPRET THE RELATIONSHIP BETWEEN SPOT, FORWARD, AND PAR RATES.
- ASSESS THE IMPACT OF MATURITY ON THE PRICE OF A BOND AND THE RETURNS GENERATED BY BONDS.
- DEFINE THE "FLATTENING" AND "STEEPENING" OF RATE CURVES AND DESCRIBE A TRADE TO REFLECT EXPECTATIONS THAT A CURVE WILL FLATTEN OR STEEPEN.
- DESCRIBE A SWAP TRANSACTION AND EXPLAIN HOW A SWAP MARKET DEFINES PAR RATES.
- DESCRIBE OVERNIGHT INDEXED SWAP (OIS) AND DISTINGUISH OIS RATES FROM LIBOR SWAP RATES.

Note: This topic covers similar discussions which are covered in properties of interest rate reading from Book 3. Hence we will keep our focus on new topics which are not discussed in Properties of interest rate reading.

10.1 PAR RATES

Par rate is the rate of coupon which results into bond price equal to par value (face value). To understand this concept we will start with a normal bond pricing and then we will see how replacing coupon rate results into par rate.

Assume a bond paying 10% coupon rate every six months with time to maturity equal to 1.5 years. To price the bond we will use following spot rates.

Maturity T	Spot rate	Discount factor
0.5 years	5%	0.9756
1 year	6%	0.9426
1.5 year	7%	0.9019

$$Bond\ Price = \frac{C}{2} X\ 0.9756 + \frac{C}{2}\ X\ 0.9426 + \frac{C}{2}\ X\ 0.9019 + 100\ X\ 0.9019$$

Coupon rate of 10% is paid, hence

$$Bond\ Price = \frac{10}{2} X\ 0.9756 + \frac{10}{2}\ X\ 0.9426 + \frac{10}{2}\ X\ 0.9019 + 100\ X\ 0.9019 = 104.29$$

Solving this equation gives,

Bond Price = $104.29

Lets take P as par rate which results into bond price equal to par value $100.

$$\frac{P}{2} X\ 0.9756 + \frac{P}{2}\ X\ 0.9426 + \frac{P}{2}\ X\ 0.9019 + 100\ X\ 0.9019 = 100$$

In the above equation we replaced C with P and price of bond 104.29 with 100.

Hence par rate is that coupon rate which gives the bond price equal to its face value or say par value.

We can further solve this equation into,

$$\frac{P}{2}(\ 0.9756 + 0.9426 + 0.9019) + (100\ X\ 0.9019) = 100$$

Solving this equation gives

P = 6.957%

The semi annual coupon rate of 6.957% will result into bond price equal to $100. Hence, 6.957% is the par rate.

Upon solving the above equation we get closed form equation for calculating par rate in %

$$P = \frac{2\ X\ 100\ x(1-d(T))}{A(T)} = \frac{2\ X\ 100\ x(1-0.9019)}{2.820} = 6.957\%$$

Where A(T) is annuity factor = 0.9756 + 0.9426 + 0.9019 = 2.820, sum of all discount factors

d(t) is discount factor at time t = 0.9019,

We can also use par value and coupon rate for bond pricing as follows,

$$V = 100 + \frac{C - p}{2} A(t) = 100 + \frac{10 - 6.957}{2} 2.820 = \$104.29$$

The price of bond is same using standard method and par rate method. For exam purpose, remember above formula which we need to use in case of bond pricing using par rate.

10.2 IMPACT OF MATURITY ON PRICE OF A BOND

Maturity of the bond affects bond prices depending on term structure changes and value of forward rate agreement.

- If the forward rate agreement has a positive value if the coupon is greater than the forward rate for the final period.
- IF the forward rate agreement has a negative value if the coupon is less than the forward rate for the final period.

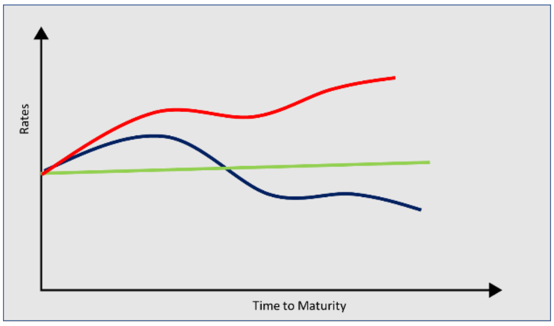
10.3 PROPERTIES OF SPOT, FORWARD AND PAR RATES

Key properties of spot, forward and par rates.

- For flat term structure, all par rates and all forward rates equal the spot rate.
- If the term structure is upward-sloping, the par rate for a certain maturity is below the spot rate for that maturity.
- If the term structure is downward-sloping, the par rate for a certain maturity is above the spot rate for that maturity.
- If the term structure is upward-sloping, forward rates for a period starting at time T are greater than the spot rate for maturity.

10.4 RATE CURVES

Rate curve can be found in three forms, upward slopping, downward slopping and flat (Check following diagram). Term structure is upward slopping when short term rates are lower compared to long term rates. Upward slopping term structure is normally found in market. Downward slopping term structure is when short term rates are higher than long term rates. And flat term structure is when both long and short term rates are at same level.



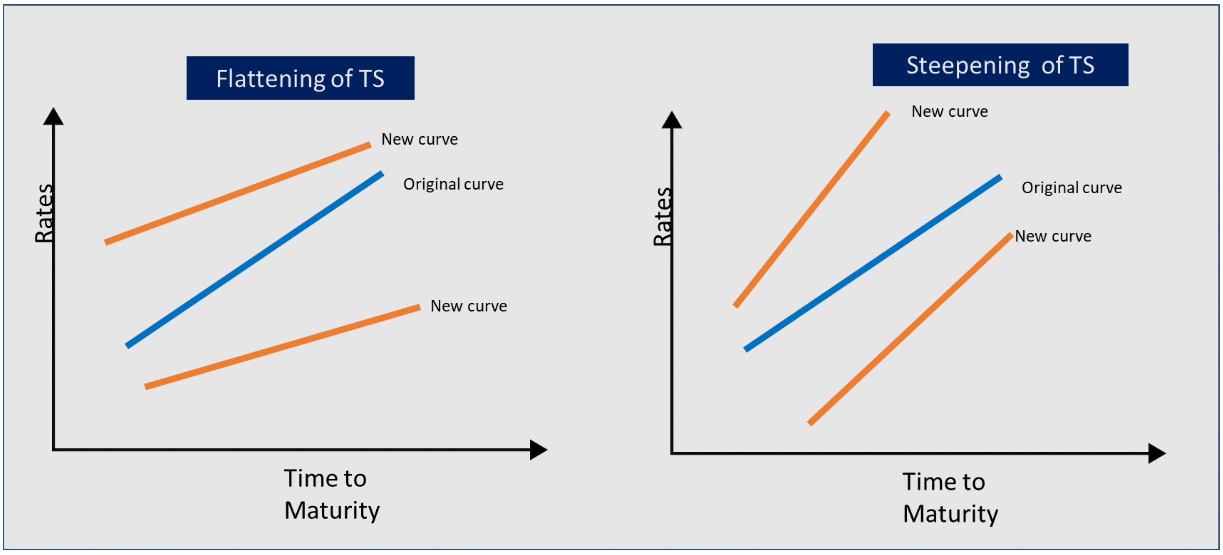
Flattening and steepening of term structures

Flattening of TS:

- When short term maturity rates move up more compared to long term rates.
- Or short term rates moves down less compared to long term rates.

Steepening of TS:

- When short term rate moves down more compared to long term rates.
- When short term rate moves up less compared to long term rates.



Impact on investors decision in flattening and steepening of term structure.

Assume an investor takes long position in long term rate bond and short position in short term rate bonds. When the term structure flattens, long term rates will decline and hence long term bond's price will increase. On the other hand short term rates increase means short term bond's price will decline. Increase in bond prince for long term rate bond will generate profits for investor with long position in long term rate bond's and decrease in price of short term rate bonds means investor with short position in short term rate bond will gain because of this decline.

Reverse is true for investor holding long position in short term bonds and short position in long term rate bonds and term structure steepens. Long term rate bond's price will decline and short term rate bond's price will increase which will result in profit.

10.5 SWAP AND SWAP RATES

Swap and swap rates are discussed in Book 3 Reading swaps. Just to summarize this topic in this section, swap is the transaction between two parties exchanging fixed rate for floating rate on notional principle. The fixed rate is assumed to be zero sum game which accounts for applicable Libor rates. If there is no change in Libor rates then there is no gain or loss to both the parties. However, Libor rates do change and hence it generates cash flow for one of the parties at every settlement date.

Swap rate is the fixed rate of swap transaction. Swap rate is also considered as par rate of bond because this rate ideally should result into par value which will ensure the value of swap is zero at inception. We know the notional is never exchanged in swap transaction. However, if we assume notional is exchanged in the end by both the parties, then each leg acts as a bond. Fixed leg can be treated as fixed coupon paying bond and floating leg can be considered as floating rate paying bond.

To conclude we can say Swap rates are used to determine par rates and par rates can be used to determine spot rates.

10.6 OVERNIGHT INDEX SWAP (OIS)

Overnight index swaps are swaps for short duration swaps based on overnight index rate. In OIS floating rate is geometric average of overnight fed fund rates for that period. OIS rate is the rates are fixed rates in OIS swap. Similar to Libor swap rates are used to determine Libor term structure (as discussed in previous section), OIS rates are used to determine OIS term structure. OIS settles on quarterly basis and typical duration of swap can range from one week to one year.

Reading 11 Bond Yields and Return Calculation

AFTER COMPLETING THIS READING YOU SHOULD BE ABLE TO:

- **DISTINGUISH BETWEEN GROSS AND NET REALIZED RETURNS, AND CALCULATE THE REALIZED RETURN FOR A BOND OVER A HOLDING PERIOD INCLUDING REINVESTMENTS.**
- **DEFINE AND INTERPRET THE SPREAD OF A BOND, AND EXPLAIN HOW A SPREAD IS DERIVED FROM A BOND PRICE AND A TERM STRUCTURE OF RATES.**
- **DEFINE, INTERPRET, AND APPLY A BOND'S YIELD-TO-MATURITY (YTM) TO BOND PRICING.**
- **COMPUTE A BOND'S YTM GIVEN A BOND STRUCTURE AND PRICE.**
- **CALCULATE THE PRICE OF AN ANNUITY AND A PERPETUITY.**
- **EXPLAIN THE RELATIONSHIP BETWEEN SPOT RATES AND YTM.**
- **DEFINE THE COUPON EFFECT AND EXPLAIN THE RELATIONSHIP BETWEEN COUPON RATE, YTM, AND BOND PRICES.**
- **EXPLAIN THE DECOMPOSITION OF THE PROFIT AND LOSS (P&L) FOR A BOND POSITION OR PORTFOLIO INTO SEPARATE FACTORS INCLUDING CARRY ROLL-DOWN, RATE CHANGE, AND SPREAD CHANGE EFFECTS.**
- **EXPLAIN THE FOLLOWING FOUR COMMON ASSUMPTIONS IN CARRY ROLL-DOWN SCENARIOS: REALIZED FORWARDS, UNCHANGED TERM STRUCTURE, UNCHANGED YIELDS, AND REALIZED EXPECTATIONS OF SHORT-TERM RATES; AND CALCULATE CARRY ROLL DOWN UNDER THESE ASSUMPTIONS.**

11.1 REALIZED RETURNS OF A BOND

Realized return of a bond is return earned by an investor on a bond investment without adjustment for time value of money. Realized return is calculated by comparing investment value and final value of a bond(with coupons earned if any). First we will see realized return of the bond without reinvestment of coupon.

$$Realized\ return = \frac{BV_t - BV_{t-1} + C_{t-1}}{BV_{t-1}}$$

Where,

BV_t = final value of a bond.

BV_{t-1} = Purchase value of bond

C_{t-1} = Coupons earned

Illustration 1:

Assume an investor purchased a bond at $98 and final value of the bond at the end of 6 months at $100 and coupon earned is $5. To calculate return earned

$Realized\ return = \frac{100-\ 98+\ 5}{98} = 7.14\%$

With reinvestment:

When the coupon is received at the end of say 6 months in investor invest this coupon at market rate. Then the return earned on the investment of this coupon is also considered in the realized calculation.

Illustration 2:

Assume an investor purchased a bond at $98 and the final value of the bond at the end of 1 year is $100. Investor earned coupon of $5 at the end of 6 months and at the end of 1 year. Investor invested the coupon at the semi annual risk free rate of 6%.

$Realized\ return = \frac{100-\ 98+\ 5+(5\ X1.03)}{98} = 12.39\%$

In the above calculation 5 X 1.03 is to calculate the coupon value after adjusting reinvestment at 6% semi annual rate.

Realized return rate of 12.39% is annually compounded earning rate. This can also be converted into semi annual return or continuously compounded rate using the concepts learned in reading on Properties of interest rate. (This topic is covered in TI BA II Plus calculator course available on website for free).

Net return after financing cost

Assume in the above case investor borrowed funds to invest in the bond at 3% annual rate. Hence

$Realized\ return = \frac{100-\ 98+\ 5+(5\ X1.03)-(3\%\ X\ 98)}{98} = 9.40\%$

11.2 SPREADS

The calculated price of the treasury securities and market price may be different. The reason for this difference is the spread. Spread is the rate which explains the difference in pricing of the bond theoretical price and actual market price of the bond. Lets assume forward rates with semi annual compounding given in following table

Time Period	Forward rate	Discount factor (Ref note 1)
0 to 6 months	0.7%	0.996512
6 to 12 months	1.2%	0.990569
12 to 18 months	1.6%	0.982707
18 to 24 months	2.0%	0.972977

Note 1: Discount rate is calculated using the method which we learned in previous readings. For example discount factor of 12 to 18 months is the factor applied to discount cash flow at the end of 18th month which calculated after considering the effect of all the previous forward rates.

Discount factor of 18th month = $\frac{1}{\left(1+\frac{0.007}{2}\right)X\left(1+\frac{0.012}{2}\right)X\left(1+\frac{0.016}{2}\right)}$ = 0.982707

Discount factor of 24th month = $\frac{1}{\left(1+\frac{0.007}{2}\right)X\left(1+\frac{0.012}{2}\right)X\left(1+\frac{0.016}{2}\right)X\left(1+\frac{0.020}{2}\right)}$ = 0.972977

We can use these discount rates to calculate bond price. Assume a $100 face value bond paying 5% semi annual coupon with 24 months maturity left. To calculate the price of this bond we can simply

Bond Price = 2.5 X 0.996512 + 2.5 X 0.990569 + 2.5 X 0.982707 + 102.5 X 0.972977 = 107.155

Now assume the investor is able to buy this bond at market price of the bond is $105.50. To convert this difference into spread we need to increase forward rate by spread. The value of spread will give us the market price of the bond. Assume S is the spread of the bond, then Bond pricing after adjustment for spread is

$$105.50 = 2.5\left(\frac{1}{(1+\frac{0.007}{2}+\frac{S}{2})}\right) + 2.5\left(\frac{1}{(1+\frac{0.007}{2}+\frac{S}{2})(1+\frac{0.012}{2}+\frac{S}{2})}\right)$$
$$+ 2.5\left(\frac{1}{(1+\frac{0.007}{2}+\frac{S}{2})(1+\frac{0.012}{2}+\frac{S}{2})(1+\frac{0.016}{2}+\frac{S}{2})}\right)$$
$$+ 102.50\left(\frac{1}{(1+\frac{0.007}{2}+\frac{S}{2})(1+\frac{0.012}{2}+\frac{S}{2})(1+\frac{0.016}{2}+\frac{S}{2})(1+\frac{0.020}{2}+\frac{S}{2})}\right)$$

S is the value which gives the bond price equal to 105.50. As the bond price decreases, the spread increases.

Exam note: In exam you are not expected to calculate spread of the bond because it is cumbersome process. Understanding the concept of spread and its impact on bond price is sufficient enough from exam perspective.

Spread can be calculated for various bonds like spread of AAA rated bond and Treasury Bond. The spread depends on maturity of the bond which can be seen in the above calculation setup. We have detailed discussion on Spread in FRM part II Book 2 Credit Risk.

11.3 YIELD TO MATURITY (YTM)

Bonds yield to maturity is the yield earned by investor till maturity of the bond. When the Bonds yield which is the single discount rate is applied to bonds cash flow, it gives the present value equal to market price. Lets take the same example given in spread discussion(previous section), where market price of the bond paying semi annual coupon of 5% is 105.50. To calculate YTM of the bond,

$$105.50 = 2.5\frac{1}{(1+Y/2)} + 2.5\frac{1}{\left(1+\frac{Y}{2}\right)^2} + 2.5\frac{1}{\left(1+\frac{Y}{2}\right)^3} + 102.50\frac{1}{\left(1+\frac{Y}{2}\right)^4}$$

Solving this equation for Y we will get YTM of the bond. Please note, the bond price is the market price and not theoretical price. Solving Y using the equation is cumbersome task and hence we prefer using TI BA II plus calculator. To calculate YTM in calculator, please follow these steps

Clr > TVM , 4 > N , -105.50 > PV, 100 > FV, 2.5>PMT , CPT > I/Y

The result on the calculator is 1.08 which is periodic yield of 1.08%. To convert this into Yield to Maturity we need to multiply it by total periods in a year because YTM is annual rate. In our case total periods in a year are 2. Hence YTM of the bond is 1.08 X 2 = 2.17%.

YTM assumes the coupons are reinvested and all the bonds cash flows till maturity are realized.

Properties of Yield to Maturity

Following are the basic properties of YTM (must remember for exam)

- When YTM = Coupon rate, Bond trade at par
- When YTM < Coupon rate, bonds trade for more than its par value and as the bond reaches to maturity its value declines towards the par value (assuming constant YTM).
- When YTM > Coupon Rate, bonds trade for less than its par value and as the bond reaches to maturity its value increases towards the par value (assuming constant YTM).

11.4 ANNUITIES AND PERPETUITY

Annuities are the fixed income securities which pays fixed amount for specified periods. Unlike bonds, annuities do not pay any principal at maturity. For example, a security pays $100 at the end of every year for 10 years. To calculate the present value of this security assuming YTM of 5%, (Using TI BA II plus calculator)

Clr > TVM , 10 > N , 5 > I/Y, 0 > FV, 100>PMT , CPT > PV

We get present value equal to -722.173. Negative sign indicates the cash flow opposite to of PMT. In our case we assumed cash inflow of $100 every year, hence -722.173 indicates the cash outflow of $722.173.

Note: We can also calculate present value of annuity using formula which is pretty complex and not useful for exam as well as in real life work.

Perpetuity, are the payments received forever (unlimited years) on periodic basis at the rate C. For example, if a security pays $10 on semi annual basis and YTM is 5% then its worth can be calculated as

$$Value\ of\ security = \frac{C}{Y} = \frac{5}{0.025} = 200$$

The difference between annuities and perpetuity is, annuity pays cash flow for certain period and perpetuity pays cash flow for unlimited years.

11.5 THE EFFECT OF COUPON

Bond with same maturity and different coupons may have different YTM. The fact that the bonds with same maturity and different coupons have different YTM is called coupon effect. The effect of the coupon can be summarized as

As the coupon increases, the average time until the maturity decreases and the spot rates applicable for early payment dates become more important in YTM calculation.

- For upward sloping term structure, the spot rates for the early payment dates are lower than the spot rates for the final payment date. Hence, YTM declines as the coupon rate increases.
- For downward sloping term structure, the spot rates for the early payment dates are higher than the spot rates for the final payment date. Hence, YTM increases as the coupon rate increases.

Please note, yield is not the reliable measure of relative value of bond. A bond with higher coupon rate may have lower yield than the bond with lower coupon. This is the result of time to maturity and slope of term structure.

11.6 JAPANESE YIELDS

In Japan Yields are quoted on a simple yield basis, This means these is no compounding in the yield measurement. Consider a bond with a face value of $100 and T years to maturity. Bond is currently priced P and pays coupon C. The Japanese yield is calculated as

$$Y = \frac{C}{P} + \frac{100 - P}{PT}$$

For example, 10 year bond has coupon of 5% and the price is 102. The Yield is

$\frac{5}{102} + \frac{100-102}{102\ X\ 10} = 0.047 = 4.7\%$

Hence the yield is 4.7%.

11.7 CARRY ROLL DOWN

Carry roll down estimates the return achieved if there is no change to factors relating to interest rate. The common assumptions when calculating carry roll down is forward rates are realized. Realized forward rate means the forward rates using in bond pricing are same as the real spot rate at that time. Please note, forward rates are based on current term structure and rates are rarely realized because term structure changes constantly. But realized forward rate assumes the forward rates for a future period are unchanged as we move forward in time.

There are carry roll down assumptions other than forward rates realized,

- Unchanged term structure
- Unchanged yield curve

11.7.a Assumption: Forward rates are realized

As we previously discussed, forward rates are realized means the forward rates remain unchanged as we move through time. Carry roll down is the difference in bond price from one period to another holding this assumption true. Following table provides the forward rate and realized rate holding the realized forward rate assumption (Table is same as provided in GARP curriculum book).

	At Time T0	At Time T0.5 (6 months after)
Period	Forward rate % Semi annual	Rate realized after 6 months
0-6 months	0.7	
6-12 months	1.2	1.2
12-18 months	1.6	1.6
18 – 24 months	2.0	2.0
Bond Price at T0 and T0.5 for bond paying 2.5% coupon	102.226*	101.334**

*Calculation is done for 4 periods and in ** calculation is done after 6 months period is over and hence calculation is done for remaining life of the bond which is 3 periods. Calculations are done using bond pricing methods which we learned before.

In this table we can see forward rates are realized after six months. This shows if the forward rates are realized then bond price changes from $ 102.226 to $ 101.334 after 6 months. After 6 months coupon of $1.25 will be received, hence carry roll down is

USD 101.334 + 1.25 – 102.266 = 0.358.

Alternate quicker way of calculating carry roll down assuming rates are realized is to assume the return earned on any bond over the next period is always the prevailing one period rate. In our example rate earned in first 6 months is 0.7% semi annually compounded. The return earned in 6 months period is 0.35%. Using the current price of the bond and return earned,

Carry roll down = 0.0035 x 102.226 = 0.358 (same as above)

Carry roll down (quicker method) applies to every portfolio where forward rates are realized irrespective of size of the portfolio.

Implications in trading strategy:

- If investor expects forward rates to be realized, both long term bonds and short term bonds provide same returns.
- If investor expects realized rates < forward rates, Long term bonds will provide better gross return.
- If investor expects realized rates > forward rates, short term bonds will provide better gross returns.

11.7.b Assumption: Unchanged Term Structure

Following table provides the forward rate at time T0 and forward rate at time T0.5 assuming term structure unchanged.

	At Time T0	At Time T0.5 (6 months after)
Period	Forward rate % Semi annual	Unchanged TS after 6 months
0-6 months	0.7	
6-12 months	1.2	0.7
12-18 months	1.6	1.2
18 – 24 months	2.0	1.6
Bond Price at T0 and T0.5 for bond paying 2.5% coupon	102.226	101.983

The carry roll down = 101.983+1.25 – 102.266 = 1.007

The arguments in favor of unchanged term structure assumption is it that an upward sloping

TS reflects the investors risk preferences because investors demand extra return to induce them to invest for long maturities. Assuming the investors risk preferences are not expected to change, the term structure should retain its shape.

11.7.d Assumption: Unchanged YTM

The assumption is the one period gross return, assuming the yield remains unchanged, is yield itself. A criticism of this assumption is that we don not normally expect the yield on the bond to remain unchanged. In an upward sloping term structure environment, we expect the yield of a coupon bearing bond to increase as we approach the bond's maturity. In a downward sloping term structure environment, we expect yield to decrease as we approach maturity.

11.8 P&L

Profit and loss from a fixed income portfolio can be split into several components as follows

- The carry roll down:
- Rate changes:
- Spread Changes:

Suppose the bond considered earlier that provide a coupon of 2.5% is trading for USD 101.5. The market price of the bond is 102.226. Hence investor earned spread of say 36.6 basis points per year.

P&L can be decomposed as

Initial Bond price	101.50
Carry roll down	0.54
Rate Changes	0.30
Spread changes	0.10
Final Value of bond	101.19
Cash – carry (Coupon)	1.25

In this case the gain is

101.19 + 1.25 – 101.5 = 0.94

P&L decomposition is then split into

- A carry roll down of 0.54
- The impact of a term structure change of 0.30 and
- Spread change of 0.10

0.94 = 0.54 + 0.30 + 0.10

Reading 12 Applying Duration Convexity and DV01

LEARNING OBJECTIVES

- DESCRIBE A ONE-FACTOR INTEREST RATE MODEL AND IDENTIFY COMMON EXAMPLES OF INTEREST RATE FACTORS.
- DEFINE AND COMPUTE THE DV01 OF A FIXED INCOME SECURITY GIVEN A CHANGE IN YIELD AND THE RESULTING CHANGE IN PRICE.
- CALCULATE THE FACE AMOUNT OF BONDS REQUIRED TO HEDGE AN OPTION POSITION GIVEN THE DV01 OF EACH.
- DEFINE, COMPUTE, AND INTERPRET THE EFFECTIVE DURATION OF A FIXED INCOME SECURITY GIVEN A CHANGE IN YIELD AND THE RESULTING CHANGE IN PRICE.
- COMPARE AND CONTRAST DV01 AND EFFECTIVE DURATION AS MEASURES OF PRICE SENSITIVITY.
- DEFINE, COMPUTE, AND INTERPRET THE CONVEXITY OF A FIXED INCOME SECURITY GIVEN A CHANGE IN YIELD AND THE RESULTING CHANGE IN PRICE.
- EXPLAIN THE PROCESS OF CALCULATING THE EFFECTIVE DURATION AND CONVEXITY OF A PORTFOLIO OF FIXED INCOME SECURITIES.
- DESCRIBE AN EXAMPLE OF HEDGING BASED ON EFFECTIVE DURATION AND CONVEXITY.
- CONSTRUCT A BARBELL PORTFOLIO TO MATCH THE COST AND DURATION OF A GIVEN BULLET INVESTMENT, AND EXPLAIN THE ADVANTAGES AND DISADVANTAGES OF BULLET VERSUS BARBELL PORTFOLIOS.

12.1 INTRODUCTION

In this reading we will discuss concepts such as DV01, duration and convexity. These are one factor risk measures which assumes the interest rate TS movements are mainly due to single factor. One factor assumes, yield changes impact bond price. In the next topic we will discuss multiple factors which impact bond price.

We will also cover detailed discussion of bond duration and convexity which is the part of Book 3 Properties of interest rates as per curriculum. In this reading we will combine discussion from both the readings.

12.2 DV01

The DV01 is the measure of dollar value change in fixed income security price for one basis point change in interest rate (yield). In DV01 DV stands for dollar value and 01 is indicative of 1 basis point change. The one percent (1%) is 100 basis points and hence 1 basis point means 0.01%. In fraction terms 1 Basis point is equal to 0.0001. DV01 in formula is represented as

$$DV01 = -\frac{\Delta Price\ of\ security}{\Delta Change\ in\ yield}$$

For exam purpose to calculate the DV01 we use TI BA II Plus calculator.

Illustration:

Lets assume a 10 year $100 Face value bond paying semi annual coupon of 2% is currently trading at $91.41 (assuming 3% yield). To calculate DV01 of this bond we can simply increase or decrease yield by one basis point and recalculate the price of bond. If we take bond yield of 3.01% (1basis point increase) we will get bond price equal to $91.334. The difference $0.066 ($91.344 – 91.41) is duration of bond.

Please note, the bond price and interest rate changes are inversely related, When the interest rate increases bond price decreases. To compensate for this negative relationship, we use negative sing in formula of DV01. The DV01 of a bond is always positive due to setting formula. However, DV01 of portfolio can be negative or positive depending on the long or short position in portfolio. For long portfolio DV01 is positive and for short portfolio its negative. You can also think like, when you are long portfolio, you own DV01. Because DV01 is positive, owning it will result in positive DV01. On the other hand, when you short portfolio, DV01 is negative (opposite of owning).

DV01: Impact of basis point increase vs decrease.

Suppose a portfolio consists of Treasury Bonds with face value of USD $1 Million paying 10% coupon semi annually. The Value of a bond is 1037922.03. If the rate increases by 5 bps, the value of the bond is then $1036594.52. We can estimate the DV01 as

$$DV01 = \frac{1037922.03 - 1036594.52}{5} = 265.50$$

Now if we assume, the decrease in interest rate by 5 basis points, the value of portfolio after decrease is $1039251.68. DV01 in this case is 265.93.

This shows us the DV01 for increase in interest rate is different from the decrease in interest rate basis points. DV01 is less for same basis point increase compared to same basis point decrease.

There are multiple variations of DV01. We can assume the basis point change in spot rates, bond yield or forward rates. In short the any rates which we used for bond pricing in previous reading, can be tweaked to calculate DV01. When DV01 is yield based it is called as yield based DV01, for spot rate change it is called as DPDZ and for forward rate changes it is called as DPDF. Irrespective of names underlying concept is same, i.e. changing rate by 1 basis point to check the impact on bond price. Please note, change in one basis point change in rate means 1 bps change in term structure.

- Yield-based DV01: The change in price from a one-basis-point increase in a bond's yield.
- DVDZ or DPDZ: The change in price from a one-basis-point increase in all spot (i.e., zero) rates.
- DVDF or DPDF: The change in price from a one-basis-point increase in forward rates

Note: In the following discussion we will use direct portfolio values for understanding without showing detailed calculation which is not important for exam. GARP showed full bond calculation for every basis point change in interest rate in curriculum book. However, these calculations are not important because we use calculator functions for bond pricing and not the formulas. If you are interested in checking full calculation, please check GARP curriculum book.

12.3 HEDGING USING DV01

This concept is same as hedging we learned in reading related to hedging using futures. First we need hedge ratio and then we calculate face value of hedge instrument.

$$Hedge\ ratio = \frac{DV01(portfolio)}{DV01\ (Hedge\ instrument)}$$

$$face\ value\ of\ hedge\ instrument = Hedge\ ratio\ X\ face\ value\ of\ portfolio$$

Illustration:

Assume the DV01 of a \$1 million face value portfolio is \$200. This portfolio can be hedged using another bond security which has DV01 of \$150. The face value of the bond required to create perfect hedge is

$$FV\ of\ hedge\ instrument = \$1{,}000{,}000\ X \frac{\$200}{\$150} = \$1333333$$

Hence we need short position in \$1,333,333 face value of hedge instruments to hedge portfolio. Please note, to hedge long portfolio we need short position in hedge instrument and vice versa. This is similar to hedging with futures.

12.4 DURATION

Similar to DV01 duration is the measure of sensitivity of bond price to change in bonds yield. However, DV01 is quoted in dollar terms whereas duration is measured in %. In the following section we will discuss Macaulay's Duration, Modified duration, and effective duration (or simply duration).

Macaulay's Duration (Mac durationo

Duration concept was introduced by Frederick Macaulay which was named after him as Macaulay's duration. Macaulay's duration in purpose and use is very different from the modified duration and effective duration. Macaulay's duration measures the time(or duration) taken by investor to recover his initial investment value in bond via coupon and principle paybacks. For FRM examination we are not concerned with the Macaulay's duration calculation process which is kind of lengthy process. If you are interested in learning calculation of this duration please check my Video on Falcon Edufin channel. Or use this link https://www.youtube.com/watch?v=lvMxKpQNwlM

The reason to introduce Mac duration here because modified duration is uses Mac duration in calculation.

12.4.a Modified Duration (Mod duration)

The modified duration is extension of Macaulay's duration. However, Mod duration is used to measure sensitivity of a bond to changes in interest rates. The modified duration is calculated as

$$modified\ durfation = \frac{Mac\ Duration}{(1 + yield\ to\ maturity)}$$

Let's assume, the Mac duration of a bond is 5.25 years. To calculate the modified duration of a bond assuming the yield to maturity of 6%, we can

$$Modified\ duration\ = \frac{5.25}{1.06} = 4.95\%$$

Modified duration is interpreted as, for every 1% change in interest rate, bond price will change by 4.95% in opposite direction (due to inverse relation). Please note Mac Duration is quoted in years and modified duration is quoted in % term.

In modern days, Mac duration is less relevant compared to modified duration or effective duration as it does not offer any practical use. However, Mac duration was used to develop modified duration, we need to know the basics of Mac duration.

If you ever wonder, why sensitivity of interest rate to bond is called as duration (which indicates time period) , just remember, first measure of sensitivity was developed using Mac duration and the duration part of the name was carried forward for other measures of interest rate sensitivity.

12.4.b Effective duration (or Duration)

Effective duration is approximation of modified duration which measures the sensitivity of interest rate to bond in percentage form. Duration is given by

$$Duration = -\frac{\Delta P}{P\Delta r} = -\frac{Change\ in\ bond\ price}{Bond\ price\ X\ change\ in\ yield}$$

The formula can be modified to show the change in price of bond as

$$\Delta P = \ -D\ X\ P\ X\ \Delta r$$

The negative sing indicates the relation of change in yield is inversely related to price change. Means, if the yield increases then bond price will decrease.

Illustration (duration calculation)

Assume the bond with following features is currently trading at $117.

- Coupon: 5% semi annual

- Duration: 10 years
- Yield: 3%
- Face value: $100

To calculate the duration, first we need to calculate the effect of yield change on bond price. We use approximation method by calculating bond price by first increasing and then decreasing yield.

Lets assume the yield increase and decrease by 10 bps (0.0010 in fraction)

Bond price (yield increase) = $116.23

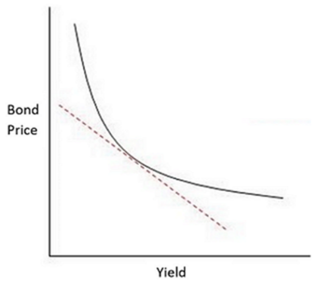
Bond price (yield decrease) = $118.12

$$\boldsymbol{Duration\ of\ bond} = \frac{\mathbf{118.12 - 116.23}}{\mathbf{117\ X\ 2\ X\ 0.0010}} = \mathbf{8.076\%}$$

Hence, if the interest rate changes by 1% then bond price will change by 8.076%. This is applicable for small changes.

12.4.c Convexity

In the previous section we discussed the duration sensitivity measure. Duration is linear sensitivity measure of interest rate. However, the bond price and yield are non linearly related. Hence if you check the effect of 1% yield change on bond price using TI BA II Plus calculator, you will find change in bond price is closer to 8.076% but with slight difference. This is because, calculator captured the actual relationship of yield and bond price (because of calculation method).



Convexity measures the curve which is not captured by duration in linear measurement. Note, Duration will measure the linear part of yield rate effect on bond price and convexity measures the non linear part of the yield rate effect on bond price not captured by duration. If we combine effect of both duration effect and convexity effect, we get the actual yield effect on bond price.

$$Convexity\ = \frac{1}{P}\left[\frac{P^{+} + P^{-} - 2P}{(\Delta r)^{2}}\right]$$

Illustration (GARP Illustration)

Consider 3 year bond paying coupon of 10% per year semi annually. The bond price is $1037992.03. Assuming the 5 basis point change in rates

P+ = 1036594.52

P- = 1039251.68

$$Convexity = \frac{1}{1037992.03}\left[\frac{1036594.52 + 1039521.68 - 2\ X\ 1037922.03}{0.0005^2}\right] = 8.246$$

The effective duration provides the impact of a small parallel shifts in the term structure. The convexity is used to accuracy of duration's linear measure.

The estimate of the price change using both bond duration and convexity is

$$\Delta P = -D\ X\ P\ X\ \Delta r + \frac{1}{2} C\ X\ P\ x\ \Delta r^2$$

Illustration

The duration of bond is 10 and convexity of the bond is 100. To estimate the effect of 100 basis point increase on change on bond price, assuming current bond price is \$102.

$$\Delta P = -10\ X\ 102\ X\ 0.010 + \frac{1}{2}\ X\ 114.8\ X\ 102\ X\ 0.010^2 = -10.2 + 0.5854 = \$9.614$$

12.5 PORTFOLIO DURATION AND PORTFOLIO CONVEXITY

The DV01 for a portfolio is simply the sum of the DV01s of the components of the portfolio.

The duration and convexity for a portfolio is weighted average of duration and convexity.

Bond No	Value \$	DV01 \$	Duration	Convexity
Bond 01	5M	20	10.5	105
Bond 02	10M	30	12.5	130
Bond 03	20M	40	13	155

Portfolio DV01: 10 + 30 + 40 = \$80

Portfolio Duration: $\left(\frac{5}{35} X10.5\right) + \left(\frac{10}{35} X12.5\right) + \left(\frac{20}{35} X13\right) = 12.5$

Portfolio Convexity: $\left(\frac{5}{35} X105\right) + \left(\frac{10}{35} X130\right) + \left(\frac{20}{35} X155\right) = 140.71$

12.6 BARBELL VS BULLET PORTFOLIO

Consider three bonds:

1. A 5-year bond with a 2% coupon,
2. A 10-year bond with a 4% coupon, and
3. A 20-year bond with a 6% coupon.

We assume that the term structure of interest rates is flat at 4% (semi-annually compounded). The effective durations and convexities of the three bonds can be calculated as indicated earlier in this chapter.

If an investor wants a portfolio with an effective duration of 8.1758, he or she can buy the ten-year, 4% coupon bonds. This is referred to as a bullet investment because only one bond is involved. An alternative is to construct a portfolio from the other two bonds with an effective duration equal to 8.1758. This is known as a barbell investment. Suppose that a proportion (3 of the portfolio is invested in the five-year bond, and a proportion (1 — (3) is invested in the 20-year bond. The duration will be 4.6764/3 + 12.6235(1 - (3)

This equals 8.1758 when /3 is 0.5597 (which we will round to 0.56).

There are therefore two ways a portfolio with a duration of 8.1758 can be created

1. Invest all funds in the ten-year, 4% coupon bond, or
2. Invest 56% of funds in the five-year, 2% coupon bond and 44% of the funds in the 20-year, 6% coupon bond.

The portfolios have the same duration but different convexities. The first has a convexity of 78.8981. The second alternative has a convexity of:

24.8208 X 0.56 + 212.4604 X 0.44 = 107.382

As illustrated by Figure 12.3, a positive convexity improves the bond holder's position when there is a parallel shift in interest rates. As convexity increases, the improvement increases.

While both strategies provide a yield of 4% and a duration of 8.1758, the barbell strategy always produces a better result when there is a parallel shift in the yield curve. The barbell strategy therefore appears to dominate the bullet strategy. We might deduce from this that there is an opportunity for arbitrageurs:

- Invest a certain USD amount in the barbell, and
- Short the same USD amount of the bullet

This would be profitable if shifts in the term structure were always parallel. However, this is not the case and non-parallel shifts do occur. In fact, the bullet investment performs better than the barbell investment for many non-parallel shifts in the term structure.

It is also worth noting that we have assumed that the yield curve is initially flat at 4% (so that the yield on all instruments is 4%). This means that if the term structure remains unchanged, all three bonds will provide a 4% return and thus the returns from the barbell and bullet investments will be the same. However, if the yield curve instead has the commonly observed upward sloping convex shape, it can be shown that the yield on the bullet investment is greater than the yield on the barbell investment.16 If yields stay the same, the bullet investment will outperform the barbell investment by the yield differential.

When researchers construct models of how the interest rate term structure moves, their objective is usually to produce what is termed a no-arbitrage model. This is a model where there are no arbitrage opportunities open to investors. Suppose a simple model is proposed where the term structure is always flat. Based on our arguments in this section, we know that a simple model where the term structure is always flat is not a no-arbitrage model.

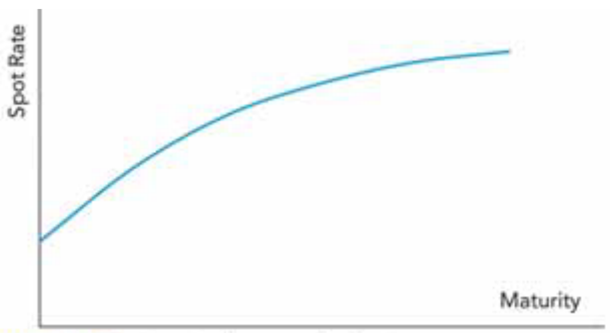


Figure 12.4 **Typical upward sloping term structures.**

12.8 CALLABLE AND PUTABLE BOND

A callable bond is a bond where the issuing company has the right to buy back the bond at a pre-determined price at certain times in the future. A company will tend to do this when interest rates have declined so that it can re-finance at a lower rate.

Consider a five-year bond that can be called after three years (but not at any other time). There is a temptation to ignore the call feature when calculating effective duration and regard the bond as a non-callable five-year bond. This is an incorrect assumption because the call feature reduces duration.

Another approach would be to assume that the probability of the bond being called remains constant. For example, suppose that when the bond is valued there is a 40% chance that the bond will be called. When calculating the effective duration, we could calculate:

- D_{called}: the effective duration of the three-year bond,
- $D_{notcalled}$: the effective duration of the five-year bond

We could then set the effective duration of the callable bond as:

$$0.40\, D_{called} + 0.6\, D_{notcalled}$$

Although this approach is better than ignoring the call feature, it would not be correct either. When interest rates increase, the probability of the bond being called is reduced. A correct approach is therefore as follows

- Value the bond today.
- Value the bond if all interest rates increase by one basis point. (This calculation incorporates the effect of the one-basis-point increase on the probability of the bond being called.)
- Calculate effective duration from the percentage change in the price.

A puttable bond is a bond where the holder has the right to demand early repayment. A puttable bond should be treated like a callable bond when calculating effective duration. In this case, the probability of the put option being exercised increases as interest rates increase.

12.9 EFFECTIVE DURATION VERSUS DV01

In choosing between effective duration and DV01, an analyst must decide whether to consider the impact of rate changes on the value of a position in dollars or as a percentage. In the first case, DV01 is appropriate; in the second case, effective duration is appropriate. DV01 increases as the size of a position increases, while effective duration does not. (If a position is doubled in size, DV01 doubles while effective duration remains the same.)

A bond investor is usually interested in returns, which are typically measured in percentage terms. This usually means effective duration is the better measure.

In other situations, DV01 may be more appropriate. For example, consider a bank that has just entered an interest rate swap. We can calculate how the value of the swap would change for a one basis point change in rates. However, effective duration is not a meaningful measure in this situation because the value of the swap is zero (or close to zero). Therefore, DV01 is likely to be the most appropriate measure for swaps (as well as interest rate futures).

Reading 13 Modeling Non-Parallel Term Structure Shifts and Hedging

AFTER COMPLETING THIS READING YOU SHOULD BE ABLE TO:

- DESCRIBE PRINCIPAL COMPONENTS ANALYSIS AND EXPLAIN ITS USE IN UNDERSTANDING TERM STRUCTURE MOVEMENTS.
- DEFINE KEY RATE EXPOSURES AND KNOW THE CHARACTERISTICS OF KEY RATE EXPOSURE FACTORS, INCLUDING PARTIAL 01S AND FORWARD-BUCKET 01S.
- DESCRIBE KEY-RATE SHIFT ANALYSIS.
- DEFINE, CALCULATE, AND INTERPRET KEY RATE 01 AND KEY RATE DURATION.
- COMPUTE THE POSITIONS IN HEDGING INSTRUMENTS NECESSARY TO HEDGE THE KEY RATE RISKS OF A PORTFOLIO.
- RELATE KEY RATES, PARTIAL 01S, AND FORWARD-BUCKET 01S AND CALCULATE THE FORWARD-BUCKET 01 FOR A SHIFT IN RATES IN ONE OR MORE BUCKETS.
- APPLY KEY RATE AND MULTI-FACTOR ANALYSIS TO ESTIMATING PORTFOLIO VOLATILITY.

13.1 MAJOR WEAKNESS IN SINGLE FACTOR APPROACH FOR HEDGING

The approach we discussed in previous reading considers only single factor and hence is very limiting because it assumes the portfolio is affected by only one factor interest rate. Also it assumes that rates of all terms change by the same amount based on one factor also known as parallel shift. Multifactor approach allows for non parallel shift by considering multiple factors for measuring the sensitivity to the portfolio.

13.2 PRINCIPAL COMPONENT ANALYSIS

PCA (Principle component analysis) is statistical technique (which we learned in Quants Machine learning topic) which can be used to understand the term structure movements in historical data. This technique observes the daily movements in rate for various maturities and identifies certain factors. These factors are term structure movement with property that,

- Daily term structure movements observed are linear combination of the factors.
- Factors are uncorrelated.
- First few factors (2 or 3) accounts for most of the daily movement.

Rate Maturity	Factor							
	1	2	3	4	5	6	7	8
1 year	−0.134	−0.385	0.786	0.465	0.002	0.002	−0.015	0.006
2 year	−0.266	−0.493	0.067	−0.599	0.568	−0.009	0.025	0.001
1 year	−0.331	−0.420	−0.129	−0.219	−0.736	0.294	0.135	−0.058
5 year	−0.410	−0.195	−0.314	0.237	−0.060	−0.589	−0.529	0.094
7 year	−0.432	0.015	−0.286	0.394	0.228	−0.074	0.709	−0.128
10 year	−0.411	0.196	−0.101	0.195	0.221	0.692	−0.346	0.311
20 year	−0.383	0.399	0.230	−0.164	−0.024	−0.005	−0.188	−0.761
30 year	−0.364	0.444	0.344	−0.317	−0.174	−0.287	0.209	0.544

The above table provides the 8 factors applicable to Treasury rates up to 30 year period. The factor loadings are the values by which each of the rates move when there is one unit of factor movement. These factors are ordered by their impact on rates. The interpretation of factor loading for factor 2 is, when there is +1 unit of Factor 2, the 2 year rate changes by 0.493 basis point. When the relationship is negative, this will create an inverse impact. When the factor 1 is positive (negative) it will push rates lower (higher). The total impact on rate on any given day is linear combination of all the factors.

The importance of factors is measured by the standard deviation of its factor scores. Factor score is variable relating to the daily changes and standard deviation. The standard deviation of all of the rate changes across all observations is the standard deviation of the factor score. The following table shows the factor score's standard deviation for each factor.

Factors							
1	2	3	4	5	6	7	8
13.58	4.66	2.32	1.54	1.05	0.82	0.74	0.66

Table 3.2

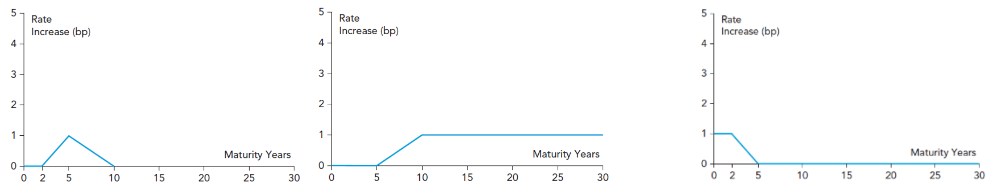
For all the data total variance is 216.44 ($13.58^2 + 4.66^2 + \cdots + 0.66^2$). in which first 3 factors account for 97.64% of the variance. The method to calculate the contribution of factors in variance is simple

$$Variance\ of\ first\ two\ factors = \frac{13.58^2 + 4.66^2}{216.44} = 95.16\%$$

This analysis shows that the one factor model considered in previous reading is an imperfect hedging tool since it only approximately reflects movement in the first factor. In this reading we consider how the metrics introduced in previous reading can be extended to accommodate a multifactor model.

13.3 PARTIAL '01 AND FORWARD-BUCKET '01

In previous reading we discussed DV01 which provides the impact of one basis point shift for all spot rates on the value of a portfolio. On the other hand, when we shift only one rate instead of all rates, we call this as key rate shifts. Assume one basis point shift in two year rate, 5 year rate and 10 year rate, the following graph shows how shift in these rates can be defined. +



The impact of shift in one rate at a time referred to as partial 01 or key rate 01(KR01).

The combined effect of the three rate shifts is one basis point shift in all rates and this way we can split the DV01 into three measures.

$DV01 = KR01_1 + KR01_2 + KR01_3$

Where $KR01_1$, $KR01_2$ and $KR01_3$ are reduction in a portfolio value from one basis point increase in 2 year, 5 year and 10 year spot rates respectively.

KR01s then can be used to hedge a portfolio in more direct way from their sensitivity to spot rates.

Partial 01s are suitable for measuring and hedging of swap portfolios. On the other hand Forward bucket 01s are similar in nature and used for hedging of swap portfolios are derived from change in shape of yield curve instead of rate shift like in Partial 01.

13.4 KEY RATE 01 AND KEY RATE DURATION

We know DV01 is equal to sum of all individual key rates KR01s. Key rate duration measures the sensitivity of portfolio value to 100bps change in yield for a given maturity. To understand this we will take the example of 10 year zero coupon bond. Please note that the risk along the curve of a 10 year ZCB is not equivalent to that of 10 year par bond due to coupon payments.

	(1) Value	(2) Key rate 01	(2) Key rate Duration
Initial value	26.22113		
2 year shift	26.22423	-.0010	
5 year shift	26.22656	-.0035	

10 year shift	26.25565	-.0345	-13.15
30 year shift	26.10120	.1219	

Above table provides the calculation of key rate DV01s and durations for 100 face amount of C-Strip. The C-Strip curve on that day was taken as the basic pricing curve with key rate shifts. First column gives the initial price of the C-strip and its present value after applying KR01s shift. Column 2 gives the key -rate 01s. Denoting the key-rate 01 with respect to the key rate Y^k as, these are defined analogously to DV01 as

$$DV01^k = -\frac{1}{10000}\frac{\Delta P}{\Delta y^k}$$

Using this formula we can get the key rate with respect to 10 year shift as

$$-\frac{1}{10000}\frac{26.25565-26.22113}{0.01\%} = -0.03452$$

In other words the Cstrip increases in price by 0.03452 per 100 face amount for a positive one basis point five year shift.

Key rate duration denoted as D^k is defined similar to key rate using formula

$$D^k = -\frac{1}{P}\frac{\Delta P}{\Delta y^k}$$

And key rate duration is calculated as

$$-\frac{1}{26.22113}\frac{26.25565-26.22113}{0.01\%} = -1.316$$

Similar principles can be applied to all the other key rates and key rate durations.

13.5 KEY RATE EXPOSURE

The exposures in Table 13.3 can be converted to exposures to the shifts in Figures 13.2, 13.3, and 13.4. Table 13.4 shows the impact of the shifts in Figures 13.2, 13.3, and 13.4 on the five spot rates in Table 13.3. For example, the shift in Figure 13.2 involves a one-basis-point shift in the one-year rate and a 0.6667-basispoint shift in the three-year rate, with none of the other rates being affected. The KR01s for the portfolio with the exposures in Table 13.3 can therefore be calculated as indicated in Table 13.5. To illustrate how a hedge position can be obtained, we will use the data in Table 13.6. This shows KR01s for a portfolio and three different hedging instruments. The positions in the three hedging instruments necessary to reduce the KR01s to zero can be calculated by solving three simultaneous equations. If x1, x2, and x3 are the positions in the three hedging instruments, the equations are

Table 13.3 Decrease in Value of Portfolio for a One-Basis-Point Increase in Spot Rates

Spot Rate Maturity (Years)	1	3	5	9	15
Portfolio Value Increase	95.62	270.26	424.35	677.93	944.74

126 + 20x1 + 3x2 + 3x3 = 0

238 + 2x1 + 22x2 + 4x3 = 0

385 + x1 + 4x2 + 25x3 = 0

The solution to these equations is x1 = -3, x2 = -8, and x3 = -14. The portfolio can therefore be hedged with short positions of 3, 8, and 14 in the three hedging instruments.

Table 13.4 Changes in Spot Rate for Changes in Figures 13.2, 13.3, and 13.4

	Spot Rate Maturity (Yrs)				
Shift	**1**	**3**	**5**	**9**	**15**
Figure 13.2	1	0.6667	0	0	0
Figure 13.3	0	0.3333	1	0.2	0
Figure 13.3	0	0	0	0.8	1

Table 13.5 KR01s for Portfolio

Partial 01	Calculation	Result
$KR01_1$	95.62 + (0.6667 × 270.26)	275.8
$KR01_2$	(0.3333 × 270.26) + 424.35 + (0.2 × 677.93)	650.0
$KR01_3$	(0.8 × 677.93) + 944.74	1,487.1

Table 13.6 Data for Hedging Using KR01s

		Hedging Instruments		
	Portfolio	**1**	**2**	**3**
$KR01_1$	126	20	3	3
$KR01_2$	238	2	22	4
$KR01_3$	385	1	4	25

13.6 FORWARD BUCKETS

In the previous section we discussed the use of Key rate duration. However, when we have multiple rates things can be come more complicated. The solution here is to create a bucket of Key rates which have similar impact on the portfolio. Forward rate can be used in buckets. Forward bucket 01s convey the exposure of a position to different parts of the curve in a much more direct way. Forward bucket 01s are computed by shifting the forward rate over each of several defined regions of the term structure, one region at a time. The term structure can be divided into buckets like 0-2 years, 2 -5 years, 5-10 years and so on. The choice of buckets depends on the purpose. If the purpose is to hedge the short term portfolio then bucket with short end is more suitable compared to long end. A swap market making desk, with business across the curve, might use the buckets defined for his section.

Note: Forward bucket calculation is not covered in this text because it is highly unlikely to get tested in exam. If you want to know more about the calculations please refer GARP curriculum book.

13.7 APPLYING KEY RATE AND MULTIFACTOR ANALYSIS TO ESTIMATE THE PORTFOLIO VOLATILITY.

It is easy to calculate the VaR and expected shortfall using factors given in the following table because these factors are uncorrelated. Standard deviation of the factor score for the ith factor and change in the value of the portfolio when there is a movement in the term structure corresponding to one unit of the factor. The equation of the daily change in the value of the portfolio is based on the first three factors (because of highest impact).

$$\sigma_p = \sqrt{\sigma_1^2 f_1^2 + \sigma_2^2 f_2^2 + \sigma_3^2 f_3^2}$$

Illustration:

- The portfolio value changes by +20 when the TS has the changes indicated by the first factor with value 13.58

- The portfolio value changes by +35 when the TS has the changes indicated by the second factor with value 4.66
- The portfolio value changes by -10 when the TS has the changes indicated by the 3rd factor with value 2.32

The standard deviation of the daily changes in the value of portfolio is

$$\sqrt{13.58^2 X\ 20^2 + 4.66^2 X\ 35^2 + 2.32^2 X\ 10^2} = 317.52$$

Assuming the normal distribution, the 10day 99% VaR can be calculated as 2336 ($\sqrt{10}\ X\ 2.33\ X\ 317.52$) and expected shortfall is calculated as 2676(Formula is complicated hence not shown here). In exam ES calculation is highly unlikely to get tested.

Duration Measure

We know duration can be calculated using DV01s using the formula

$$duration = \frac{10000\ X\ DV01}{Value\ of\ the\ portfolio}$$

We can similarly convert any of the 01 measures(discussed in this reading) into duration measures using

$$duration = \frac{10000\ X\ (01\ Measure)}{Value\ of\ the\ portfolio}$$

For example, the 0-1 year forward bucket 01 measure is 0.0102. To convert this into forward bucket duration

$$duration = \frac{10000\ X\ 0.0102}{105.6014} = 0.97$$

Similar approach can be used for KR01s and bucket 01s.

Reading 14 Binomial Trees

LEARNING OBJECTIVES

- CALCULATE THE VALUE OF AN AMERICAN AND EUROPEAN CALL OR PUT OPTION USING A ONE-STEP AND TWO-STEP BINOMIAL MODEL
- DESCRIBE HOW VOLATILITY IS CAPTURED IN THE BINOMIAL MODEL
- DESCRIBE HOW THE VALUE CALCULATED USING A BINOMIAL MODEL CONVERGES AS TIME PERIODS ARE ADDED.
- DEFINE AND CALCULATE THE DELTA OF A STOCK OPTION.
- EXPLAIN HOW THE BINOMIAL MODEL CAN BE ALTERED TO PRICE OPTIONS ON STOCKS WITH DIVIDENDS, STOCK INDICES, CURRENCIES, AND FUTURES.

14.1 INTRODUCTION TO BINOMIAL TREES

In this reading we will discuss binomial trees valuation method for option pricing processed by Cox, Ross and Rubinstein (1979) and is used widely for American option. Please note in this reading we will majorly use the European option to understand concept of binomial option pricing. However, we now have well developed models like BSM model for the European option which reduces the importance of binomial tree for European option. Because there is an inherent limitation originated due to early exercise feature of an American option, advanced models don't work well with American option.

Derivatives are valued using no arbitrage pricing. Law of no-arbitrage pricing states that if two portfolios A and B should be priced at same level if two portfolios generates same payoffs. We will also see risk neutral valuation which also relies on no arbitrage pricing.

Note: In all the following concept explanations, unless otherwise specified we will assume all the risk free rates are continuously compounded and the options European and not American.

14.2 ONE-STEP AND TWO-STEP BINOMIAL MODEL

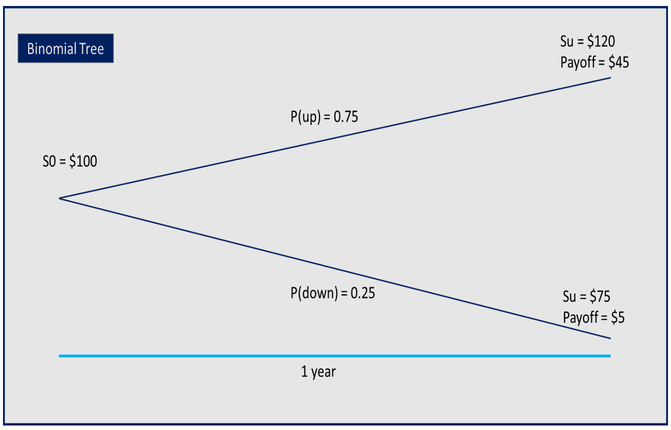
We know the simple principle of pricing any asset using expected value of the asset in future which is the present value of expected value of an asset adjusted for time value of money should be the price of an asset.

Assume a stock which has expected value of $520 at the end of 1 year, the price of stock today is the present value of expected value of the stock at the end of 1 year. Assuming the risk free rate of 10% continuously compounded the present value of $520 is $470.6. Hence, the fair price of the stock today based on the expected value calculation is $470.6. In this pricing we only need two steps,

- Calculation of the expected payoff
- Present value of the expected payoff

We can apply these principles for option pricing as well. If we know the expected payoff of an option which is nothing but the value of an option at maturity, we price this option by calculating the present value of the expected payoff of an option.

Illustration: Assume a call option on stock currently trading at $100. The time to expiration of this call option is 1 year. Stock price is expected to move up to $120 and down to $80 at the end of one year. The strike price of the call is $75 which will generate the payoff on this option of $45(120-75) and $5 (80-75) for upside and downside movement respectively at the end of 1 year. If we assume the probability of upside and downside movement is 0.75 and 0.25. Following diagram provides this information in Binomial Tree form.



In the previous readings, we discussed the concept of expected value and how the pricing of security is done with the help of expected value. In this case there is 75% probability that option will pay $45 and 25% probability that option will pay $5. Hence the expected payoff is $35 (0.75 X 45 + 0.25 X 5). The expected payoff is generated at the end of 1 year. The present value of the expected payoff is the value which an investor should be willing to pay. Assuming 10% continuously compounded risk-free rate the present value of the expected payoff is

$$PV(Payoff) = PRice\ of\ the\ call\ option = \$35\ X\ e^{-0.10\ x\ 1}\ = 31.66$$

Hence $31.66 should be the price of a call option assuming the price movements and probability of those price movements.

What do we need to learn in this chapter?

In the above illustration, we used some given information like the probability and movement of stock. For the pricing of options, we need to calculate probability of upside and downside movements and movements of stock price based on volatility. To learn binomial option pricing, we will use step by step approach –

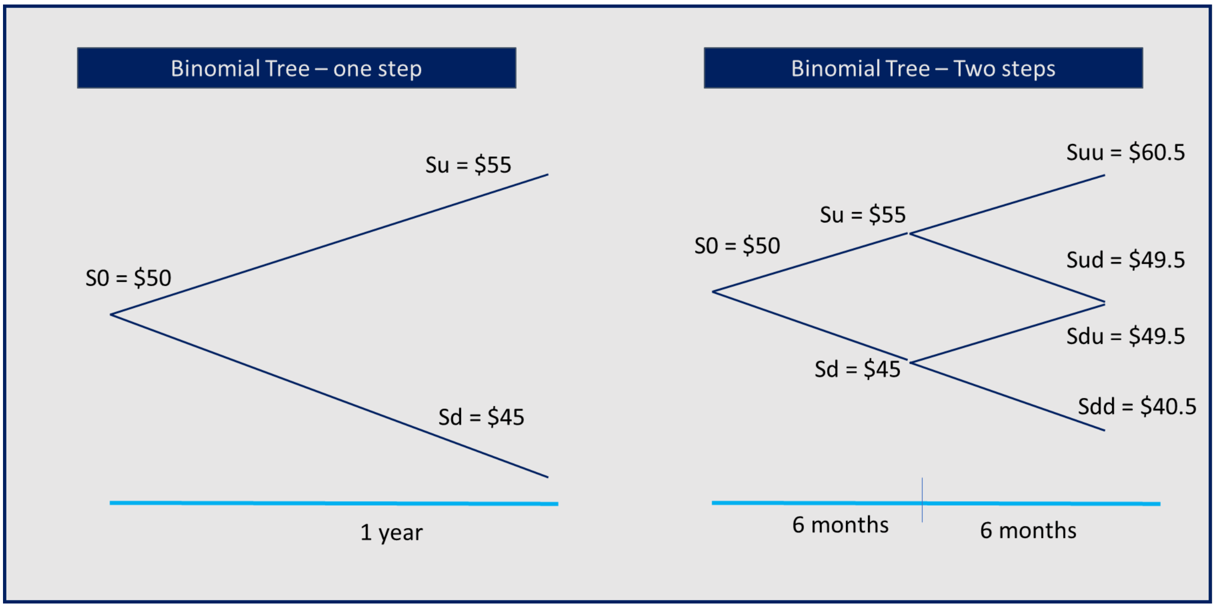
- Step 1: Deciding the steps in the binomial tree and the period of each leg.
- Step 2: Calculation of stock price movements and movement factors.
- Step 3: Calculation of probability of upside and downside movements.
- Step 4: Calculation of payoff of option at the expiration or on each leg (for American option).
- Step 5: Calculating present value of payoff to price the option.

Professors note: In the recent exams instead of asking full questions on Binomial option pricing GARP asked questions that were from intermediate steps. Hence, I intentionally decided to use step by step approach for binomial pricing explanation. At each step we will learn possible variations of that step and how it connects to next step.

14.2.a Step 1: Deciding Steps in the Binomial Option Pricing

Steps or jumps in simple terms means the total price changes instances in a binomial tree. In one step model we assume price will change only once (up and down) in life of an option. Similarly for two steps we assume price will change for two times in the life of the option. In real life with the help of computer we can use 100 steps which gives more refined answers. But from exam perspective we will focus only on one step and two step model. In recent exams GARP is more inclined towards testing one steps compared. This is mainly because option pricing using two step model is very time consuming process in exam setting but GARP asked two step questions in previous examinations (till 2018 exams). To make sure you are ready for both the scenarios we will discuss both one step and two step models.

Illustration: ABC Inc stock is currently trading at $50. We want to price the call option with 1 year maturity using one step and two step binomial option pricing model assuming stock price is expected to move up and down by 10% at each step. Following diagrams shows the binomial tree for one step model and two step model.



In the above diagram we can see for one step model, only stock price is moving only one time in life of an option however in two steps model price is moving two times in a given year and hence each step of price movement of stock in for 6 months period. If we decide to price an option using 12 steps binomial model then we will have 12 steps with each step of 1 month.

Steps are very important in binomial option pricing because the price movements and probability of up and down are calculated for each step and not for life of an option. In coming sections, we will discuss concepts by considering both one step and two step models.

14.2.b Step 2 Price movements and price movement factors

Price movement is simply the up and down movement of underlying's price at every step. Price movement factors are the calculated using price movement or directly depending on question. Price movement factors are required for probability of upward movement and downward movement.

Illustration: Assume a one-step binomial model and price of an underlying currently trading at $100 is expected to move by 20%. Let's S_u and S_d are the price at upside leg and downside leg respectively and U and D are up movement factor and down movement factor respectively.

S_u = 100 + 20% = 120

S_d = 100 – 20% = 80

U = up price / original price = 120/100 = 1.20

D = 1/U = 1/1.2 = 0.8333 --------(A)

Alternate calculation of D = down price / original price = 80/100 = 0.80 ---------- (B)

Professor's note: In above case both D calculations are correct despite having slightly different answers. This also slightly affects probability calculation. There is no clarity from GARP regarding the method to use for D calculation. As per my understating and from the exam perspective, it is recommended to use method (A) of D calculation when price movements requires use of volatility and use method (B) when price movements are directly provided. One thing to keep in mind is, the model which we use for Binomial option pricing is CRR (Cox Ross Rubinstein Model) in which U X D = 1 which is only applicable if price movements are based on volatility. If volatility is not used in calculation of price movements we are not restricted by CRR model.

Price movements can be given in question in quite distinctive styles. We will discuss all the possible styles of price movements and their treatments in following table.

Style	Language	Calculation of up and down price	Calculation of up and down factors
Direct Form (underlying price)	Stock is currently trading at $100 and is expected to move up to $120 or $80 in each period.	Not required (directly provided).	U = 120/100 = 1.20 D = 80/100 = 0.80
Direct form (factor)	Stock is trading at $100 and up move factor is 1.20.	*First we need to calculate U and D in this case.* Up price = 100 X 1.20 = 120 Down price = 100 X 0.833 = 83.3	U = given = 1.20 D = 1/1.20 = 0.833
Percentage form	Stock is currently trading at $100 and is expected to move by 20% in each period.	Up price = 100+20% = 120 Down price = 100 – 20% = 80	U = (1+0.20) = 1.2 or 120/100 = 1.2 D = 1-0.2 = 0.80 or 80/120 = 0.80
Volatility form *(Further explained below)*	Stock is currently trading at $100 and volatility of stock is 20% (annual).	$Up\ price = S\ X\ e^{\sigma\sqrt{t}}$ $= 100X\ e^{0.20\sqrt{1}}$ $= 122$ $Down\ price = S\ X\ e^{-\sigma\sqrt{t}}$ $= 100X\ e^{-0.20\sqrt{1}}$ $= 81.87$ Where t is time in leg period.	$U = e^{\sigma\sqrt{t}} = e^{0.20\sqrt{1}}$ $= 1.22$ $D = S\ X\ e^{-\sigma\sqrt{t}}$ $= 100X\ e^{-0.20\sqrt{1}}$ $= 0.8187$

Price movements in the form of volatility: This is most common form used in questions by GARP and also requires formula hence we will discuss this in more details.

Using one Step model: When the price movements are given in the form of volatility we need to start with up movement and down movement factors. Using the above example in which we assumed the period of 1 year and one step model,

$U = e^{\sigma\sqrt{t}} = e^{0.20\sqrt{1}} = 1.2214$

D = 1/ U = 1 / 1.2214 = 0.8187

Up stock price = Su = $100 X 1.2214 = 122.14

Down stock price = Sd = $100 X 0.8187 = 81.87

Using two step model: In above illustration if we use two step binomial model then period for each leg will be 6 months or 0.5 years and we will need stock price at the end of each period / leg.

$U = e^{\sigma\sqrt{t}} = e^{0.20\sqrt{0.5}} = 1.152$

D = 1 / 1.152 = 0.868

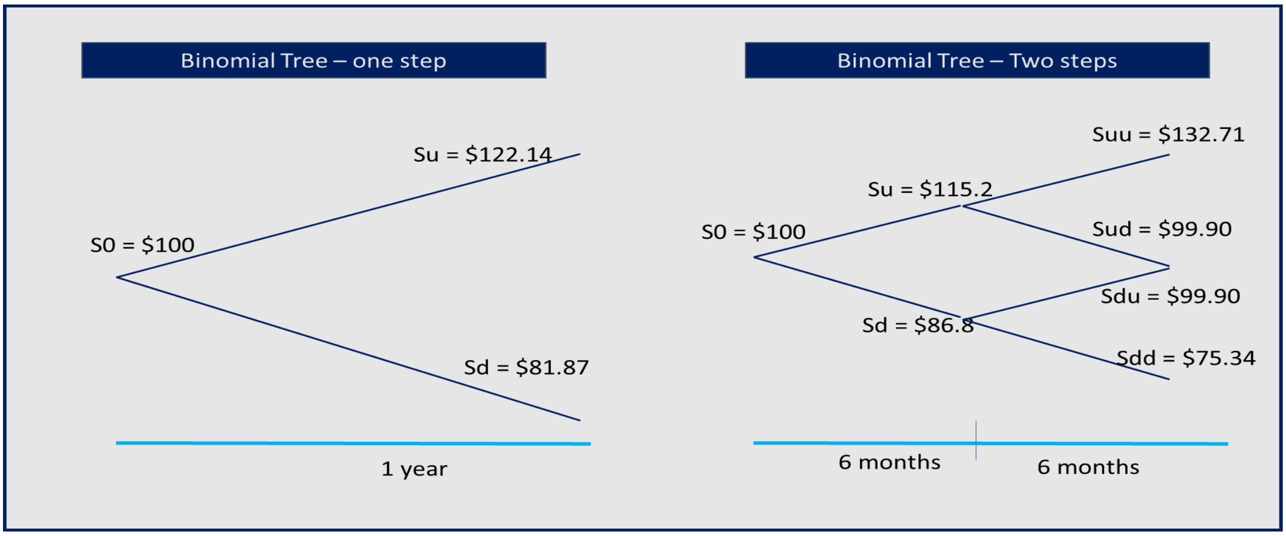
Up stock price (end of first period) =Su 1.152 X 100 = 115.2

Up stock price (end of second period) = Suu = 1.152 X 115.2 = 132.71

Down stock price (end of first leg) = Sd = 0.868 X 100 = 86.8

Down stock price (end of second leg) = Sdd = 0.868 X 86.8 = 75.34

Recombining leg = Sud or Sdu = 115.2 X 0.868 = 86.8 X 1.152 = 99.90



Once we get U and D we are ready to move into probability calculation.

14.2.c Step 3 Probability of up movement and down movement (Risk Neutral probability)

Probability is calculated using interest rates and U and D factors. Please note we calculate probability for one period and not for the life of the option. Once we get probability of up movement (Pu) and down movement, same probability is used for all up movement legs and down legs respectively. The method of probability calculation used in this case is risk neutral method and hence called as risk neutral probabilities. Risk neutral probabilities are not real world probabilities but are dummy probabilities. Risk neutral probabilities come from formula as per the Cox-Ross-Rubinstein Model.

Illustration (same as above): Stock is currently trading at $100. Annual volatility of this stock is 20%. The continuously compounded risk free rate is 10%. Calculate the risk neutral probabilities of up movement and down movement for one step and two step binomial option pricing model.

Formula

$P_u = \frac{e^{rt} - D}{U - D}$ and $P_d = 1 - P_u$

Where, Pu is probability of up movement and Pd is the probability of down movement.

Assuming one step model (using one step U and D from above calculations)

$$P_u = \frac{e^{0.10X\,1} - 0.8187}{1.224 - 0.8187} = 0.70 = 70\%$$

Pd = 1- 0.70 = 0.30 = 30% (calculations are approximate)

Assuming Two step model (using two step U and D from above calculation and time period of 0.5 year per period)

$$P_u = \frac{e^{0.10X\,0.5} - 0.868}{1.152 - 0.868} = 0.645 = 64.5\%$$

Pd = 1-0.645 = 0.355 =35.5%

Professor's note: Previously GARP asked questions on risk neutral probability calculation and in some case risk neutral probabilities were given and students were expected to calculate option price. In my opinion, this is done to save students time in exam while testing concept on this topic. You should be prepared for all the possible scenarios including full questions.

14.2.d Step 4 and 5: Option Price Calculation

We will use the same information given above for option pricing of both call and put type option.

Call option with strike price of $95 and 1 year time to maturity

One step model:

To calculate the price of an option using one step model first step is to calculate payoff. We know from above calculations(one step),

Su = $122.14 and Sd = $81.87

First we need payoff for up movement leg and down leg,

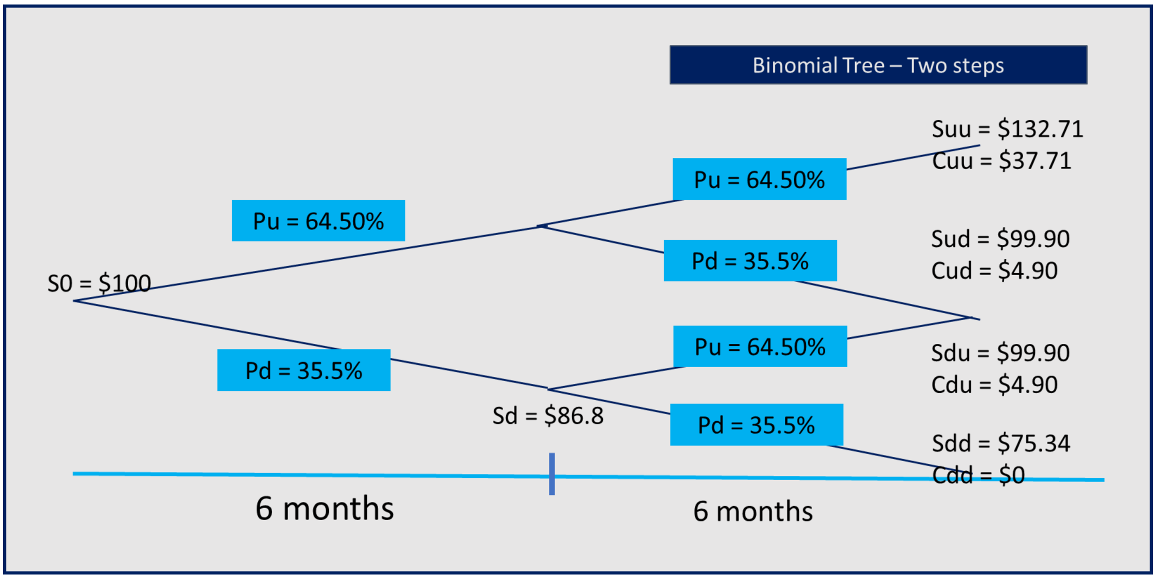
Payoff up leg = 122.14 – 95 = 27.4

Payoff down leg = 0 (call option lapsed)

Call price = present value of expected payoff

$$= \frac{27.4\ X\ P_u + 0\ X\ P_d}{e^{rt}} = \frac{27.4\ X\ 70 + 0\ X\ 30}{e^{0.10x1}} = 17.354$$

Two step model



Note: In the above diagram Cuu is payoff of call at up up move and Cdd is payoff of call at down down move. Pu and Pd are up and down probabilities respectively.

We know from the Reading Fundamentals of probability, payoff of Cuu is expected when price moves up and up again. Hence to calculate expected payoff we will apply joint probability rule.

Call price = present value of expected payoff =

$$\frac{(0.645\ X\ 0.645\ X\ 37.71) + (0.645\ X\ 0.355\ X\ 4.90) X 2 + (0.355\ X\ 0.355\ X\ 0)}{e^{0.10\ X\ 1}}$$

= 16.225

Hence, the price of call option with strike price of $95 is $16.225

Put option price with strike price $105 (two step model)

In this case, the only difference is calculation of payoff because of put option and strike price of option. The payoff of put option with strike price $105 is.

Do it Yourself: Draw binomial tree with following payoffs (two step model)

Puu = 0

Pud or Pdu = 105 – 99.90 = 5.1

Pdd = 105 -75.34 = 30.66

Put option price = the present value of payoff =

$$\frac{(0.645\ X\ 0.645\ X\ 0) + (0.645\ X\ 0.355\ X 5.1) X 2 + (0.355\ X\ 0.355\ X\ 30.66)}{e^{0.10\ X\ 1}}$$

Hence, put option price with strike price of $105 is $5.61.

14.3 ADJUSTMENTS IN BINOMIAL OPTION PRICING MODEL

Binomial tree in theory was designed for plain vanilla European option on non dividend paying stocks. However, we can modify binomial option pricing model to make it suitable for various scenarios like dividend paying stocks, American option, currency options and so on. In the following section we will discuss the modifications for these scenarios. Please note we don't need to modify complete binomial tree, only some part of the calculation is modified. Hence in the following sections we will only discuss the modification part and rest of the tree is same.

14.3.a Dividend paying option (Yield Form)

If the stock is dividend paying stock with yield q, then alternation is made in probability calculation like this

$$P_u = \frac{e^{(r-q)} - D}{U - D}$$

Pd = 1-Pu

Rest of the calculations including up and down price and up and down move factors are same as discussed previously.

14.3.b Currency Option

In currency option the risk free rate is in above section is replaced by interest rate in domestic country and interest on foreign currency is yield on foreign currency similar to q above. If we denote Rf_d and Rf_f as interest rate of domestic and foreign currency then adjustment is made in probability calculation same as above and rest of the steps are same,

$$P_u = \frac{e^{(Rf_d - Rf_f)} - D}{U - D}$$

14.3.c Futures contract

Binomial model can also be used in futures pricing. Because these contracts are costless and zero growth, we use risk free rate of 0 which makes e^{rt} equals to 1 (anything to the power 0 is 1). Hence the probability calculation is

$$P_u = \frac{1 - D}{U - D}$$

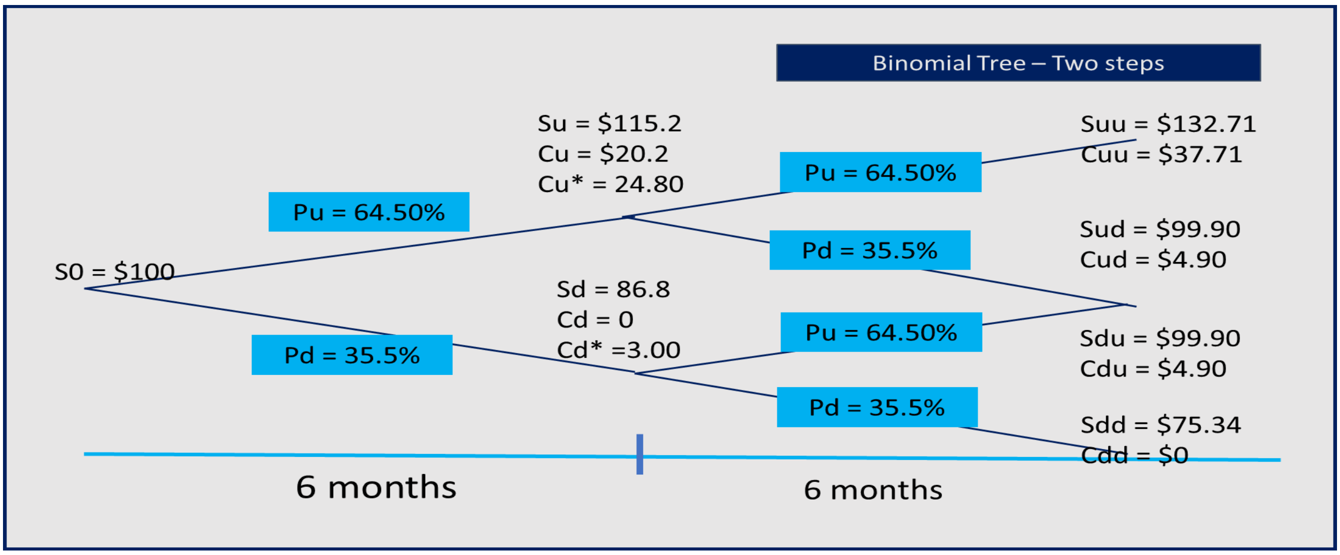
14.3.d American option

In all the earlier discussion of binomial model we assumed that the options are European options. However, we can use same binomial model for American options. In real life, Binomial option pricing model is more useful for American option than European option. This is because the recent option pricing models like BSM model which we will study in next reading are not suitable for American options due to inherent limitations of model. American options can be priced using finite difference models which is out of FRM coverage. Binomial option model when used for American options may take 10 to 100s of steps in consideration for more accurate pricing and is executed with the help of computers. Our job in this discussion is to understand how binomial trees are used for American option pricing using two step model. Because same principles are applicable in multistep binomial models.

We know American options can be exercised before the maturity date of option. Hence, we check the viability of exercise at every leg.

Illustration:

We will consider same illustration used previously with two step binomial model with only difference being the American call option instead of European Call option.



In the above diagram Cu and Cd is the payoff at the end of first leg assuming option holder exercises this option. On the other hand Cu* and Cd* is the expected value of payoff at the end of first leg by considering payoffs at end of second legs. Please check the calculations

Cu = 115.20 – 95 = 20.2 and Cd = 0 because option lapsed

$$C_u * = \frac{37.71\ X 0.6450 + 4.90\ X\ 0.355}{e^{0.10\ X\ 0.50}} = 24.80$$

$$C_d * = \frac{4.90\ X 0.6450 + 0\ X\ 0.355}{e^{0.10\ X\ 0.50}} = 3.00$$

In this case Cu indicates the value of option at end of one leg and Cu* indicates the value of option if option holder holds the option till maturity of option. Same is applicable for Cd and Cd*.

Now if the value of Cu > Cu* meaning exercising option at the end of first leg is more beneficial for option holder compared to holding it till the maturity of option. However, in our case Cu is 20.2 and Cu* is 24.80, which means holding option till maturity is more beneficial for option holder.

Hence the call price of the American option =

$$C_0 = \frac{24.80\ X 0.6450 + 3.00\ X\ 0.355}{e^{0.10\ X\ 0.50}} = 16.225$$

Hence the call price is $16.225

Let's for now assume, Cu* is 24.80 and Cu (if option exercised at the end of first leg) is $26.50. In this case option holder will exercise option before its maturity (i.e. at the end of first leg) and hence for the calculation of call price we will consider Cu and not Cu*. In this case call option price is

$$C_0 = \frac{26.50\ X 0.6450 + 3.00\ X\ 0.355}{e^{0.10\ X\ 0.50}} = 17.2740$$

Hence the call price in this case is $17.274.

Note: All the prices in all the previous calculations are theoretical prices and market price may be different from these prices.

14.4 DELTA

Delta of an option is the rate of change of call price with respect to stock price. We will elaborate more on delta in coming reading Greeks. In this reading we will discuss the use of delta. The delta denoted by Δ is the position taken in the stock to hedge a short position per unit of the derivative. Delta is the sensitivity of a derivative's value to the price of its underlying stock. Lets consider a call option which is priced at $5 when underlying price is $50 and increased to $10 when price changed to $70. To calculate the delta of call option

$$Delta = \Delta = \frac{10 - 5}{70 - 50} = 0.25$$

The delta is positive which indicates that the call price changes by $0.25 for every $1 change in stock price.

Similarly, we can also calculate the delta of put option or any other derivative. For put option delta is negative because increase in stock price decreases the option price. There is negative relationship in price of underlying stock price and option price.

14.5 RISK NEUTRAL VALUATION

An important principle in option pricing known as risk neutral valuation. Risk neutral world is the one where investors do not adjust their required expected returns for risk so that expected return on all assets is the risk free rate. This also means risk neutral investor has no preference between assets with different risks. With the risk neutral valuation principle gives the fair price for a derivatives. The binomial option pricing model we discussed before is risk neutral option pricing model.

14.6 REPLICATING PORTFOLIO FOR OPTION PRICING

Replicating portfolio method is the theory used to price the option. These portfolios consists of underlying stock, bond equal to present value of strike price and option.

Let's assume two portfolios P1 and P2.

Portfolio P1: Call Option portfolio

Portfolio P2: Stock and Borrowing (risk free portfolio)

Composition of portfolios:

Portfolio 1 Composition: Two call option with strike price of $125 and time to maturity is 1 year.

Portfolio 2 Composition: Assume stock is currently trading at $100 and expected to go up to $200 and go down to $50. To construct Portfolio P2 known as bankruptcy portfolio we need to borrow present value of $50 to buy stock. Please note stock is not fully financed by loan, we will use cash and loan of $100 to buy stock. Assuming the annual interest rate of 10%, the present value of $50 is $45.45. This means you have to spend $54.55 in cash to buy stock worth $100 ($54.55cash + $45.45 borrowing).

Price change	Portfolio 1 (2 calls with X= $125)	Portfolio 2 (S + PV(D)	Remark
Case 1: Price increased to $200	Payoff of $75 on each call and hence total payoff equals to $150	Stock is worth $200 and borrowing + interest of $50 is paid by selling stock at $200. Net payoff is $150 ($200 -$50)	We can see for both the portfolios, if the price goes up to $200, payoff is same $150.
Case 2: Price decreased to $50	Call option lapsed and hence (no) payoff is $0.	Stock is worth $50 and borrowing + interest is paid by selling $50 stock. Hence payoff is $0.	Similar to above, payoff of both the portfolios is same which is $0.

Above table shows that both the portfolios generates the same payoff at the maturity. We spend $54.55 in cash to purchase portfolio 2. We know that the two assets generating the same payoff should be priced same. Lets assume the C is call option price today, hence

Price of two call options = Price of P2 portfolio

2 X C = 54.55

C = 54.55 / 2 = $27.27

Hence the price of call option with strike price of $125 maturing in 1 year is $27.27. The price of call is depended on the current stock price.

Do it yourself:

Assume, the same case as above with current stock price of $90 instead of $100. Construct portfolios and price the call option with strike price of $125 and maturing in 1 year. Risk free rate is 10%.

Reading 15 Black Scholes and Merton Model

Learning Objectives

- Explain the lognormal property of stock prices, the distribution of rates of return, and the calculation of expected returns.
- Compute the realized return and historical volatility of a stock
- Describe the assumption underlying the Black-Scholes-Merton option pricing model.
- Compute the value of a European option using the Black-Scholes-Merton model on a non-dividend paying stock.
- Define implied volatilities and describe how to compute implied volatilities from market prices of options using the Black-Scholes-Merton model.
- Explain how dividends affect the decision to exercise early for American call and put options.
- Compute the value of a European option using the Black-Scholes-Merton model on a dividend-paying stock features and exchange rates.
- Describe warrants, calculate the value of a warrant, and calculate the dilution cost of warrant to existing shareholders.

15.1 INTRODUCTION OF BSM MODEL

In the previous reading we discussed how binomial trees can be used for option pricing. The problem with binomial trees model is that it assumes only limited number of price changes in the life of option. For example if we use two step model, it considers price change only two time in the options life. However, in reality option price keeps changing every moment or we can say continuously. Two mathematicians Fischer Black and Myron Scholes, designed a model which works considers the continuous movement of stock prices. However, this model was not closed form equation and they were facing problem in creating closed form equation. Robert C Merton is famous economist – mathematician who converted Black and Scholes model into closed form equation. Hence, the model we learn for option pricing is known as Black Scholes and Merton model. In FRM part II we will learn Merton model for credit risk measurement developed by Robert C Merton which in setup is same as BSM model, but with different input values.

Fun Facts

- Myron Scholes and Robert C Merton received Nobel prize in 1997 for BSM model. Unfortunately, Black died in 1995 before Nobel was awarded with and hence was not awarded with Nobel because Nobel prize is not awarded posthumously.
- Scholes and Merton with many others most renowned mathematicians started a fund Long Term Capital Management which went bankrupt in 1998. We will learn more about LTCM in reading Financial Disaster from Book 1.

15.2 BSM MODEL ASSUMPTIONS

Like any other closed equation model, assumptions are necessary to derive the BSM option pricing model.

Assumption	Interpretation of assumption (for Exam)
Returns are normally distributed with μ and σ constant.	Standard deviation/ volatility is used in BSM model which requires normal distribution assumption.
There are no transaction cost or taxes, and all securities are perfectly divisible.	Effect of transaction costs and taxes are not included in BSM equation directly hence adjustments are made externally (directly in calculated price).
There are no dividends on the stock during the life of the option.	BSM equation does not account for dividend payments but we can use BSM model for dividend paying stock option by adjusting stock price directly.
No arbitrage opportunities.	The price of the option is such that there is no arbitrage opportunity.
Unlimited lending and borrowing at the same risk free rate and is constant throughout time.	This is standard assumption used for using present value of strike price in input.
Option cannot be exercised early.	This is most important assumption. The implication of this assumption is that the BSM model can not be used for American option due to early exercise feature of American option.

15.3 BSM MODEL FOR EUROPEAN OPTION

As we discussed in previous section, BSM is closed form equation which takes following inputs

- S_0 = Underlying Stock Price
- X = Strike price of option
- σ = Volatility of an Option (Annual)
- r = Risk free rate
- t = Time to maturity

For pricing of option using these inputs we use,

$$Call\ price = S_0\, N(d1) - X\, e^{-rt}\, N(d2)$$

$$Put\ price = X\, e^{-rt}\,(1 - N(d2)) - S_0\,(1 - N(d1))$$

To remember these equations you can relate it with payoff formula for call and put option

In the above equation d1 and d2 are z values and N(d1) and N(d2) are probabilities calculated form Z table.

Formula for d1 and d2 (Same for call and put option)

$$d1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d2 = d1 - \sigma\sqrt{t}$$

To find N(d1) and N(d2)

Use z table and find the probability for d1 and d2.

For call option N(d1) and N(d2) are direct values from z table, however for put option we need 1- N(d1) and 1 – N(d2). Plug these values in above equation to get call and put option price.

Illustration

Compute a value of call and put option on ABC Inc using following inputs using Black Scholes and Merton model.

- ABC Inc current stock price = \$120
- Strike price of an option = \$115
- Time to maturity = 6 months
- Risk free rate = 6%
- Volatility = 12% annual

Solution:

Calculation of d1 and d2

$$d1 = \frac{\ln\left(\frac{120}{115}\right) + \left(0.06 + \frac{0.12^2}{2}\right)0.5}{0.12\sqrt{0.5}}$$

d1 = 0.89755

d2 = 0.89755 – $0.12\sqrt{0.5}$ = 0.8127

Using z table find the cumulative probability for d1 and d2

N(d1) = 0.8153

N(d2) = 0.7918

Call price = 120 X 0.8153 – 115 X $e^{-0.06*0.5}$ 0.7918 = $9.468

Put price = 115 X $e^{-0.06*0.5}$ (1-0.7918) - 120 X (1-0.8153) = $1.069

Hence, the price or value of call option is $9.468 and put option is $1.069

FAQs By students asked on this topic

FAQ 1: Do I need to remember BSM formulas

Ans: Yes, BSM formula is very important in FRM part I because GARP might ask some part of this setup like only calculation of d1, d2 or in some cases d1 d2 given and you need to calculate call or put price. Hence be to make sure you are prepared for any scenario we must remember these formulas. Also, in FRM part ii we will discuss the concept of Merton model which is same as d1 formula, hence remembering it in FRM P1 will save pain in FRM Part 2.

FAQ 2: How can I manage time if I get BSM question in exam?

Ans: Based on my understanding GARP is aware of this problem of time management in this case and hence chances are very slim that you will get full BSM question in exam (Based on previous exam trends). However, you might see question based on part of this concept in exam as we discussed in previous FAQ, hence it is recommended to learn full concept and practice full concept questions. In Falcon mock test you will see full as well as part questions on this concept.

15.4 ADJUSTMENTS IN BSM MODEL

BSM formula can be tweaked to adjust various factors like dividend limited by core model assumptions discussed before.

15.4.a Dividend

Dividend can be in continuous yield form and dollar form. Depending on the case we need to make the adjustment in stock price S0 call it Sa and use Sa in BSM model instead of S0.

Illustration on continuous Dividend Yield (Same illustration as above with dividend adjustment)

Compute a value of call and put option on ABC Inc using following inputs using Black Scholes and Merton model.

- ABC Inc current stock price = $120
- Strike price of an option = $115
- Time to maturity = 6 months
- Risk free rate = 6%
- Volatility = 12% annual
- Dividend yield = 2%

Before we proceed to BSM model we need to adjust stock prices for dividend.

$$S_a = S_0 e^{-qt}$$

Where q is the dividend yield and t is time to maturity of an option.

$$S_a = 120e^{-0.02*0.5} = 119.01$$

Now we will use Sa in BSM equation by keeping all the other factors same,

$$d1 = \frac{\ln\left(\frac{119.01}{115}\right) + \left(0.06 + \frac{0.12^2}{2}\right)0.5}{0.12\sqrt{0.5}}$$

d1 = 0.7999

d2 = 0.7999 – $0.12\sqrt{0.5}$ = 0.7150

N(d1) = 0.7881

N(d2) = 0.7627

Call price = 119.01 X 0.7881 – 115 X $e^{-0.06*0.5}$ 0.7627 = $8.674

Put price = 115 X $e^{-0.06*0.5}$ (1-0.7627) – 119.01 X (1-0.7881) = $1.265

Illustration Dividend in Dollar terms instead of yield form

Compute a value of call and put option on ABC Inc using following inputs using Black Scholes and Merton model.

- ABC Inc current stock price = $120
- Strike price of an option = $115
- Time to maturity = 6 months
- Risk free rate = 6%
- Volatility = 12% annual
- Dividend = $2 paid at the end of 2nd month and 4th month

Similar to above illustration we will adjust stock price for dividend. To adjust dividend in stock price we need present value of dividend.

Present value of dividends

Paid at 2nd month = 2 X $e^{-0.06x2/12}$ = 1.98

Paid at 4th month = 2 X $e^{-0.06 \text{ X } 4/12}$ = 1.96

Sa(adjusted stock price) = 120 – 1.98 – 1.96 = 116.06

Now we will use this adjusted stock price in BSM model

$$d1 = \frac{\ln\left(\frac{116.06}{115}\right) + \left(0.06 + \frac{0.12^2}{2}\right)0.5}{0.12\sqrt{0.5}}$$

d1 = 0.504

d2 = 0.504 – $0.12\sqrt{0.5}$ = 0.419

N(d1) = 0.6929

N(d2) = 0.6624

Call price = 116.06 X 0.6929– 115 X $e^{-0.06*0.5}$ 0.0.6624= $6.485

Put price = 115 X $e^{-0.06*0.5}$ (1-0.6624) – 119.01 X (1-0.6929) = 2.0258

15.4.b Adjustment for currency options

Adjustment in currency option for BSM model is same as dividend yield adjustment. The interest rate of domestic currency is taken as risk free rate r and foreign currency is taken as q (similar to dividend yield). Rest of the calculation is same as above.

15.4.c Adjustment for options on futures

We can use BSM model for options on futures which is similar to options on stocks. We need to make two adjustments in this case. Because underlying is futures in this case we use F (futures price) in the model instead of S (stock price). Because the futures price is the price in future (at the expiration of an option). We need to adjust futures price for time value of money. Please note, this adjustment is not required in case of S0 because S0 price at current point in time.

Hence replace S0 with F X e^{-qt} , where q is risk free rate and t is time to maturity of option on futures.

Black model also known as Black 76 model is the model which can be used to price the options on futures and forwards. Also, a call/put option on spot price of an asset is the same as call/put option on the forward price when,

- They have the same strike price and time to maturity and
- Forward contract matures at the same time as the option.

This result allows Black's model to be used to value an option on the spot pric of an asset in terms of the forward price of the asset.

15.5 WARRANTS

Options on a company's own stock are called warrants. In the event that warrants are exercised, the firm will issue additional shares, and the warrant holder will purchase those shares from the company at the strike price. The number of shares that a corporation has issued does not change as a result of an option traded on an exchange. However, a warrant enables the acquisition of additional shares at a discount from the going market rate. Existing shares' value is reduced as a result.

If markets are efficient, the potential dilution from outstanding warrants will be reflected in the share price and won't need to be taken into account when those warrants are evaluated. As a result, once warrants are publicly issued, they can be priced similarly to exchange-traded options.

A firm that is deciding whether or not to issue warrants might be interested in calculating the cost of the warrants to its existing shareholders. If there are N existing shares, and the company is contemplating the issue of M warrants (with each giving the warrant holder the right to buy one new share), it can be shown that the cost of each warrant to existing shareholders is

$$\frac{N}{N+M} X\ price\ of\ warrant$$

15.6 STOCK PRICE RETURN DISTRIBUTIONS

The underlying model used for BSM model takes the assumption of lognormally distributed stock prices, such that

$$LnS_t = N\left[Ln\, S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{t}\right]$$

Above equation shows that ln of St is normally distributed, we can say St is lognormally distributed.

This is developed to define the distribution of return. Continuously compounded annual returns are normally distributed as

$$Return\, St = N\left[\left(\mu - \frac{\sigma^2}{2}\right), \frac{\sigma}{\sqrt{t}}\right]$$

Using the property of lognormal distribution the expected value of St E(St) is

$$E(S_t) = S_0\, e^{\mu T}$$

Return calculation

The realized return R from the stock in time T

$S_t = S_0 e^{RT}$

So that

$R = \frac{1}{T} \ln \left(\frac{S_T}{S_0}\right)$

This is normally distributed with mean

$\mu - \frac{\sigma^2}{2}$

And standard deviation

$\frac{\sigma}{\sqrt{T}}$

In the model, we are assuming the expected return over a very short period is μ.

15.7 VOLATILITY

Historical return: Volatility measurement using historical stock prices is same as we discussed in reading on Basic Statistics. Volatility per year is estimated using the

$$\sigma = \sqrt{\frac{\sum(R - \bar{R})^2}{N - 1}}$$

Where, R return is calculated using $R = \ln\left(\frac{S_t}{S_{t-1}}\right)$ when the returns are continuously compounded.

Implied volatility

Implied volatility of an option is implied by BSM model. In the BSM model out of 5 inputs, 4 inputs risk free rate, strike price, underlying stock price and time to maturity are observed in market. However, volatility is not observed value in the market. Using the call and put prices available in the market, volatility can be calculated. Because this volatility calculation is implied by BSM formula, hence referred as implied volatility.

Calculation: There is no close ended formula available for calculation of implied volatility. This is iterative process where volatility is used as an input in the BSM model using systematic approach and the input which gives the call value as per BSM equal to market price of the same call is implied volatility. There are methods given in the GARP curriculum to provide inputs for implied volatility which are not very relevant in today's computer world. If you want to know about such methods please refer page number 195 Book 4 of GARP curriculum. For exam purpose we need to know if the input volatility is too high it results into higher BSM model option value compared to market, hence input volatility needs to be adjusted downwards.

Implied volatility can be calculated for American options valued using the binomial tree approach. As we discussed in previous readings, CBOE has developed indices such as SPX VIX index that tracks volatility of 30 day option on the S&P 500. Implied volatility vary with strike prices. This statement makes the foundation for Volatility smiles (topic covered in FRM Part II Market Risk).

Reading 16 Option Sensitivity Measures, The "Greeks"

LEARNING OBJECTIVE

- DESCRIBE AND ASSESS THE RISKS ASSOCIATED WITH NAKED AND COVERED OPTIONS POSITIONS.
- DESCRIBE THE USE OF A STOP LOSS HEDGING STRATEGY, INCLUDING ITS ADVANTAGE AND DISADVANTAGE AND EXPLAIN HOW THIS STRATEGY CAN GENERATE NAKED AND COVERED OPTIONS POSITION.
- DESCRIBE DELTA HEDGING FOR AN OPTION, FORWARD AND FUTURES CONTRACT.
- COMPUTE THE DELTA OF AN OPTION.
- DESCRIBE THE DYNAMIC ASPECTS OF DELTA HEDGING AND DISTINGUISH BETWEEN DYNAMIC HEDGING AND HEDGE AND FORGET STRATEGY
- DEFINE AND CALCULATE THE DELTA OF PORTFOLIO.
- DEFINE AND DESCRIBE THETA, GAMMA, VEGA AND RHO FOR OPTION POSITIONS, AND CALCULATE THE GAMMA AND VEGA FOR A PORTFOLIO.
- EXPLAIN HOW TO IMPLEMENT AND MAINTAIN A DELTA-NEUTRAL AND A GAMMA-NEUTRAL POSITION.
- DESCRIBE THE RELATIONSHIP BETWEEN DELTA, THETA, GAMMA AND VEGA.
- DESCRIBE HOW PORTFOLIO INSURANCE CAN BE CREATED THROUGH OPTION INSTRUMENTS AND STOCK INDEX FUTURES.

16.1 INTRODUCTION

Financial institutions dealing in option derivatives need to manage its risk. An institution manage risk of option using various factors.

In the reading Properties of Option we discussed factors which impacts options prices. In that reading our discussion was limited to general impact of factors on option price. In this reading we will use Greek letters measuring the impact of these factors. Following are the Greek letters and factors discussed in this reading

Greek letter	Symbol	Factor
Delta	δ	Measures impact of stock price changes on option
Gamma	Γ	Measures the rate of change of delta with respect to stock price
Theta	θ	Measure the time decay of an option
Vega	υ *or* κ	Measure the impact of volatility on option
Rho	ρ	Measure the impact of interest rate on option

Note: In Greek letters there is no such letter called Vega. The actual Greek letter used for volatility measurement is Kapa denoted by K. However, in financial markets Vega is more common term than Kapa. In exam if you see question on Kapa, you must remember the question is about Vega. In this reading we will use letter Vega for volatility.

16.2 OPTIONS HEDGING TECHNIQUES

Before we begin the discussion on Greek letters lets first understand the use of Greeks. An option trader can manage risk on option positions using following methods.

- Covered positions or Do nothing strategy
- Stop loss strategy
- Greek letters

16.2.a Covered position

When the trader keeps the option position as it is without any hedging is called as naked position also known as do nothing strategy. Assume a trader takes the short position in an option at strike price of $100. The do-nothing strategy is suitable if the underlying price at the maturity is below $100 because trader keeps the premium earned on short option. However, if the option is exercised then trader will generate loss on the position. As an alternative to naked position, trader can opt for covered position. Covered position is the position when trader also takes the position in underlying for hedge.

- Trader with short call option can cover this position by taking long position in underlying stocks. In this covered position the loss on short call option is compensated by increase in stock price.
- Trader with short put option can cover this position by taking short position in underlying stocks. In this covered position the loss on short put option is compensated by decrease in stock price.

The problem with both naked and covered position is that these strategies are not perfect and might generate losses for trader.

16.2.b Stop loss strategy

We already discussed the stop loss in Book 3. Stop loss is simple concept in which trader sets the limit to square off his position. In stop loss, if the loss increases above the desired level, the stop loss is triggered, and trader takes the opposite position which squares off the position of trader. Stop loss is not the proper hedging technique because with stop loss trader needs to bear certain losses on the position.

16.2.c Greeks

Some traders prefer sophisticated hedging techniques like static or dynamic hedging with Greeks. These techniques involve calculating measure such as delta, gamma and Vega. In the following section will discuss each Greek measure one by one.

16.3 DELTA AND GAMMA

Note: In the following discussions, unless otherwise specified, call means European call on non-dividend paying stock and put means European put on non-dividend paying stock.

The Delta (δ) of an option is the rate of change of the option price with respect to change in underlying asset. Let's assume a call option is trading at $10 when the underlying is trading at $100. When underlying price increases to $110, option price increased to $15. The delta of an option is calculated as

$$\delta = \frac{C_t - C_{t-1}}{S_t - S_{t-1}} = \frac{15 - 10}{110 - 100} = \frac{5}{10} = 0.5$$

Hence the delta of an option is 0.5. Delta of 0.5 means when the stock price changes by small amount the option price changes by 50%. For example if the stock price increases by $1 then option price will increase by $0.5. We used the word "small change" in the above statement which is very important. Because, delta measures of change only valid when the change in underlying is small.

Delta of call vs Delta of put

We already know when the underlying price increases (decreases) call price also increases (decreases). Because there is positive relationship in stock and option changes the delta of call option is positive and ranges between 0 to 1. For put option, when the underlying price increases (decreases) the put option price decreases (increases). The price change relationship of stock and put option is negative. Hence the delta of put option is negative and ranges between -1 to 0.

Key takeaways

- Delta of call – Always positive – Range 0 – 1
- Delta of put – Always negative – Range -1 to 0

16.3.a Position Delta

Position delta simply means the total delta component of position taken by a trader. Position delta can be of a single position or portfolio.

Assume, a trader takes the long position in 1000 call options. The delta of this call option is 0.75.

Position delta = 1000 X 0.75 = 750

As we know delta can be positive or negative. Similarly, the position delta can also be position or negative but in a different way. The positive or negative position delta depends on two things, the type of option (call or put) and the position (long or short).

The position delta is very complicated discussion because it requires consideration to multiple factors. So, here is the trick to simplify the position delta. Think of a long position in an option as "I own" position and short position in an option as opposite (in sign) of this position. Then we will simply apply rule of multiplication with sign.

Delta	Position Sign	Net position sign
+ (call)	+ (Long)	+ X + = +ve
+ (Call)	- (short)	+ X - = -ve
- (Put)	+ (Long)	- X + = -ve
-(Put)	- (Short)	- X + = +ve

Please note, you will own the position delta with sign of delta. Meaning if you take long put and as we know the delta of put is negative, so position delta will be negative. We will elaborate more on this with following examples -

- You are long on 100 calls with delta of 0.25. The position delta is +25 because positive delta is owned by taking long position.
- You are short on 1000 calls with delta of 0.35. The position delta is -350 because delta is positive but position is short (hence opposite sign).
- You are long on 500 put options with delta of -0.15. The position delta is -75, because delta is negative and you own it because of long position.
- You are short of 100 put options with delta of -0.90. The position delta is +90, because delta is negative and position is short.

16.3.b Portfolio Delta

Portfolio delta is simply the sum of individual position delta of in a portfolio. In the following table we will see different portfolios and their portfolio delta.

Portfolio 1		Portfolio 2		Portfolio 3 (Do it yourself)	
Option	Position Delta	Option	Position Delta	Option	Position Delta
Option 1	500	Option 1	300	Option 1	2500
Option 2	-250	Option 2	-25	Option 2	-100
Option 3	-75	Option 3	-75	Option 3	+500
Option 4	-150	Option 4	-800	Option 4	+25
Portfolio Delta	25	Portfolio Delta	-600	Portfolio Delta	

We can put portfolio delta in formula

$$\delta Portfolio = \sum \delta_i, where\ \delta_i\ is\ delta\ of\ individual\ option$$

16.3.c Delta hedging or Delta neutral portfolio

We know that the delta is risk of change in option price with respect to underlying price change. If the delta of an option is zero, then the option is risk less option. However, the mostly options not delta neutral. Hence, we create a portfolio which is delta neutral by adding some other positions with option. Generally, we can combine option with option or option with stocks to make the delta neutral portfolio.

Professors note: Creation of delta neutral portfolio is very confusing process, hence I will use very unconventional and my own method which I consider most effective for understanding of delta neutral portfolio creation.

To create delta neutral portfolio we need to know few points –

The basic rule of making delta neutral portfolio is, bring equal amount of opposite sing of delta in a portfolio which makes the portfolio delta neutral.

Delta of stock is always 1 if you take long position in a stock it brings +1 delta in portfolio and if you take short position in a stock then it brings -1 delta. (please note this statement is just given only for exam purpose and to make your question solving easy)

Illustration

Let's assume the current position delta of a option portfolio which you hold is +1500. Please note once you get the position delta, the composition of portfolio is irrelevant for delta neutral portfolio construction. This means, for all call portfolio or all put portfolio or mix of call and put portfolio with position delta of +1500, the process of creating delta neutral portfolio is same.

We have following options to make the portfolio delta neutral

Option 1: Stocks

Option 2: Call option having delta equal to 0.25

Option 3: Put option having delta equal to -0.65

Creation of delta neutral portfolio

Option 1:

We know stocks have delta of 1 and to hedge the portfolio with position delta of +1500 we need to bring delta of -1500 to make delta neutral portfolio.

Hence by taking short position in 1500 stocks which will bring -1500 delta will make delta neutral portfolio (+1500 position delta + (-1500 delta from short stock)).

Option 2:

Call having delta of 0.25. To bring -1500 delta using this call, we will need

$$Short\ calls = \frac{1500}{0.25} = 6000$$

Hence 6000 short calls of delta 0.25 are required to make the portfolio delta neutral.

Option 3:

Put option having delta of -0.65. To bring -1500 delta using this call, we will need

$$Long\ put = \frac{1500}{0.65} = 2307$$

Hence we need to take 2307 long puts to make portfolio delta neutral.

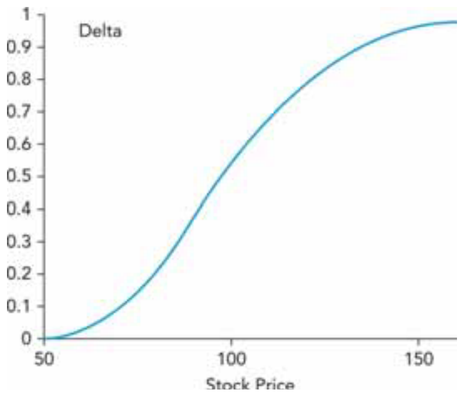
Delta neutral portfolio = Delta 0 = 1500 position delta – 0.65 X 2307 puts = 0

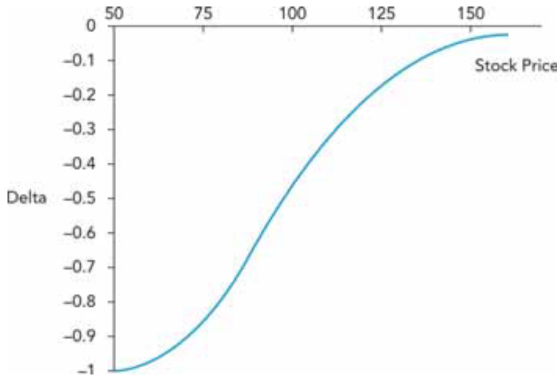
16.3.d Delta based on moneyness of the option

In the previous section we discussed that the delta ranges between 0 to 1 or -1 to 0. The value of delta depends upon the moneyness of the option.

- Delta of a call approaches to 1 as the call goes deeper in the money and approaches to 0 as the call is more out of the money. (Image in the left)
- Delta of a put approaches to -1 as the put goes deeper in the money and approaches to 0 as the put is more out of the money. (Image in the right)

Image source: GARP Curriculum Book





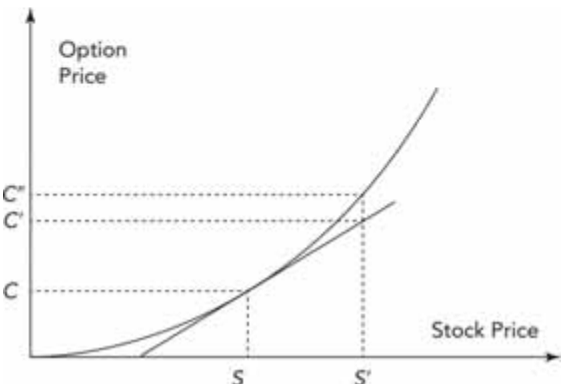
16.3.e Delta Calculation method

In BSM reading we learned how to calculate d1 and N(d1) in BSM model. The N(d1) is nothing but delta of an option and can be used for all the purposes relating to delta. In exam if you see question asking you to calculate delta of an option and the information given in the questions are inputs required for BSM model, simply calculate N(d1) using same method learned in previous method.

N(d1) = Delta of an option

16.4 GAMMA

Gamma is the rate of change of delta with respect to stock price. Mathematically speaking Gamma is partial derivative of the portfolio with respect to asset price. In general terms gamma provides the impact of stock price change on delta. The relationship of Delta and gamma is exactly like relationship of duration and convexity (from previous readings of this book).



Delta captures a linear relationship between the option price and stock price. However, the relationship between these two is nonlinear and convex. Hence gamma captures the convexity of this relationship. Because delta does not capture the true relationship of stock price and option price, creation of delta neutral portfolio will not give us perfect hedge. Hence, we use gamma and delta both to create delta gamma neutral portfolio.

When the gamma is low, the delta change is slow, and adjustment needed to keep the portfolio hedged are less frequent. However, when the gamma is high (negative or positive) the adjustments are more frequent.

Calculation of gamma is given by

$$\Gamma = \frac{N'(d_1)}{S_0\sigma\sqrt{T}}$$

We don't need to worry about this formula and calculation for exam purpose. Even if it looks simple, its calculation is pretty complicated for exam environment. In exam gamma will be given and you are expected to create a delta gamma neutral portfolio.

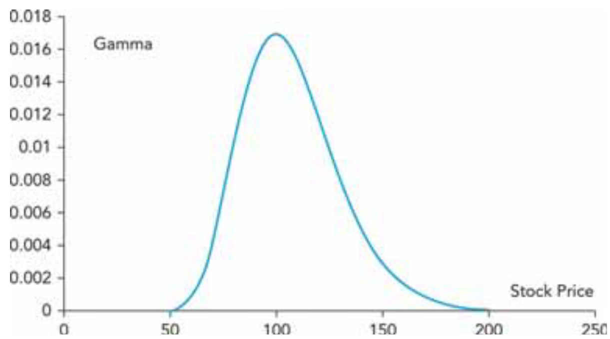
Change in the value of portfolio when an option position is delta neutral and the stock price change by ΔS is approximated by

$$\frac{1}{2} X\ \Gamma X (\Delta S)^2$$

Suppose the gamma of the position is -0.50 and the stock price increases by \$2, the value of of the option position will decrease by ½ X -0.50 X 2^2 = \$1.

Key points to note about gamma

- Gamma is always positive irrespective of the option type call or put.
- For Long position call or put gamma is positive and for short position call or put gamma is negative.
- The negative gamma is due to position (similar to position delta) and not due calculation. If you look at formula of gamma, there is not scope for negative value due to formula setting.
- Gamma is highest for at the money options and approaches zero as the option moves deep in the money or deep out of the money.



Delta gamma neutral portfolio

Delta gamma portfolio is slightly complicated process but helps in creating perfectly hedged portfolio. In this process we need to start with making portfolio delta neutral first, then make the portfolio gamma neutral and then readjust the portfolio to make it delta neutral.

Few things to note

- Delta can be neutralized by stocks or options.
- Gamma can only be neutralized by options and not by stocks because stocks have zero gamma.
- To make the delta neutral portfolio, which is already gamma neutral, the only option stocks can be used.

Illustration

Following table provides the information delta and gamma of 1 portfolio, put and stock. Using the put and stock create a delta gamma neutral portfolio.

Particulars	Portfolio	Put option	Stock
Delta	665	-0.335	1
Gamma	24	0.024	0
Vega		109.348	

To create a delta gamma neutral portfolio we need to follow following steps

Step 1: *Make the portfolio gamma neutral.*

Position gamma is 24 and to make the portfolio gamma neutral using put option having gamma of 0.024 we need to take

$$\frac{24}{0.024} = 1000\ put\ option$$

We should short 1000 put option to introduce -ve 24 gamma (Short put 1000 X gamma 0.024).

Step 2: *make portfolio delta neutral with stocks*

Before we make the portfolio delta neutral we need to assess the current delta of the portfolio after adding 1000 short puts.

Existing delta 665 + 0.335 X 1000 = 1000.

Delta of put will have positive (additive) impact on portfolio because of short put position.

To make the portfolio delta neutral we need to take short position in 1000 stocks to bring negative delta in portfolio.

Hence To make the delta neutral portfolio we need 1000 short put options and 1000 short stocks.

Note: In exam you might get part of this question like after gamma adjustment what is the left-over delta which doesn't require further calculation after step 1.

16.5 VEGA

The changing volatility throughout the time increase the risk for derivatives. Please not this is contrary to the assumption of BSM model which assumes the constant volatility. We know increase in volatility increases the option price. Hence the trader with short position in is always at risk of increase in volatility. Vega (Kapa) is the measure of volatility and calculated as

$$vega = \frac{\delta c}{\delta \sigma} = \frac{change\ in\ derivativeprice}{change\ in\ volatility}$$

A long position in option has a positive vega (similar to gamma). The vega of a European call or put option on a non dividend paying stock is given by a formula (not important for exam due to calculation difficulty).

$$vega = S_0\sqrt{TN'(d1)}$$

Vega is greatest for options at the money and tends to zero as the option moves away from the ATM (i.e. for out of the money and deep in the money option). Using the formula above, vega is expressed in decimals. Assume an option with vega of 0.40, which means the 1% change in in volatility will result in 0.40% change price of an option.

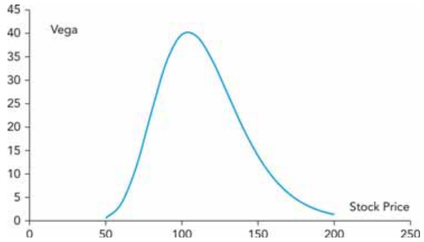


Illustration: Consider a call option on stock worth USD 100 and strike price is \$110, the risk free rate is 5% an the volatility is 25%. The vega for this option is 40 (stock price USD 100 X 0.40)

Hence the loss on a portfolio that has sold options on \$1 million shares is USD 400,000 per 1% increase in volatility. The 1% increase in volatility means volatility increased to 26% from 25%.We can also use BSM model to calculate the loss by simply changing volatility in BSM model form 25% to 26% and resultant change in option is the loss for short position holder.

Hedging vega risk is not an easy task like hedging delta risk and can not be hedged using underlying asset. This is because adding underlying asset in portfolio does not affect the vega of the portfolio of derivatives. Vega hedging requires another option on same underlying (with different strike price or maturity).

Illustration

Assume a delta neutral portfolio with vega of -ve 500. To make the portfolio vega neutral using an option with delta of 3 and vega of 2.5, trader needs to take the long position in 200 such options(-500 + 200 X 2.5 (vega) = 0). This is similar to delta hedging where we take the position in option which brings the quantity of delta in portfolio but with opposite sign.

Taking the position in option will impact the delta of portfolio and will generate delta risk. In this case the position delta after adding new option position in portfolio for hedging of vega is 600 (delta of 3 X 200 options long). The portfolio can be made delta neutral by taking short position in underlying stocks.

16.6 THETA

. The theta is the rate of change price of an option with respect to time. Theta can be calculated using some complicated mathematics which we don't need to worry about. The theta is negative for a long position in option and lose value as the time passes. However, there is certainty about the time laps in theta (i.e we know for sure with each passing day options maturity will reduce by one day) however same is not applicable for volatility or interest rate. Hence, traders can accurately measure the time decay of an option with the help of theta.

Key statement to remember:

When theta is highly negative, gamma tends to be highly positive and vice versa.

16.7 RHO

The Rho is the measure of change in price of an option with respect to change in interest rate (rf). Unlike delta, gamma or vega, for trader uncertainty of an interest rate is not very important.

16.8 RELATIONSHIP OF DELTA THETA AND GAMMA

The BSM model can be used to show the relationship of delta, theta and gamma, as

$$theta + rS_0 X delta + \frac{1}{2}\sigma^2 S_0^2 X\ gamma = r\ X\ Option\ value$$

When the portfolio is delta neutral the equation is reduced to

$$theta + \frac{1}{2}\sigma^2 S_0^2 X\ gamma = r\ X\ Option\ value$$

16.9 PORTFOLIO INSURANCE

Portfolio insurance allows the asset holder the upside potential at the same time compensating for the downside movements by generating payoffs when the portfolio value going down. Consider a portfolio of stock and as a portfolio insurance, trader takes position in put options on the same underlying. The alternative to taking position in put option is index futures contract. A trader can take short position in index futures in a proportion to insure portfolio. Traders prefer using futures for hedging compared to options due to lower costs.